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To Bundle or Not to Bundle: Firms' Choices under Pure Bundling

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ABSTRACT *We examine the economic implications of pure bundling under the settings of monopoly and duopoly. We show that under monopoly and pure bundling of goods with independent demands, the bundled price is strictly less than the sum of the unbundled prices. In the setting of duopoly and Nash prices, we examine whether bundling can be used as a tool to deter entry. In contrast to the findings of previous studies, we show that with low entry costs, entry is deterred by unbundled as opposed to bundled sales. With high entry costs, however, the incumbent chooses to bundle.*

Key Words: Pure Bundling; Monopoly; Duopoly; Entry Costs.

JEL classifications: D42; L11; L40.

1. Introduction

Previous research on the economics of commodity bundling can be segregated into three broad groups: explanations of bundling as a means of foreclosure or entry deterrence (e.g., Whinston, 1990; Carlton and Waldman, 2002; Choi, 2002; and Nalebuff, 2004), explanations of bundling as a tool for price discrimination (Adams and Yellen, 1976; Schmalensee, 1982; McAfee *et al.*, 1989, among others) and finally a cost-based theory of bundling (Salinger, 1995; Evans and Salinger, 2005). Our paper supports the findings of the second group. While prior research has generally examined both pure bundling (i.e., goods sold only as a bundle) and mixed bundling (i.e., goods are sold separately *and* as a bundle), we examine only the effects of pure bundling on a firm's optimal prices and profits under heterogeneous consumer preferences.

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There are several reasons for examining pure bundling. First, pure bundling is ubiquitous in the marketplace, and it is particularly prevalent in computer-technology related products and in information-goods industries. Examples of pure bundles include cable TV channel and satellite radio channel packages, Factiva or Dow Jones subscription service to a collection of information sources, and bundled software suites such Microsoft Office. Second, mixed bundling is rarely subject to antitrust scrutiny[1] because, under mixed bundling, the bundled price is constrained to be less than or equal to the sum of the individual prices; otherwise, consumers would never buy the bundle. By contrast, pure bundling is often referred to as 'tying' and has been viewed critically by antitrust agencies. Third, previous studies that have argued that bundling is entry-deterring have done so primarily in the setting of pure bundling. In this context, we focus not only on pure bundling's impact on consumer prices but also on whether it deters entry.

In section 2, we examine the effects of pure bundling on prices and profits under monopoly. We obtain the following results: (1) Under pure bundling of goods with independent demand[2], the bundled price is strictly less than the sum of the individual prices without bundling; (2) Profits are strictly higher with pure bundling than with unbundled sales. Previous research has derived similar results regarding the price effect of bundling by assuming demand complementarity between goods (e.g., Davis and Murphy, 2000) or bundling-induced cost efficiencies (e.g., Salinger, 1995); our paper does not rely on these assumptions. Our result on the effect of bundling on profits builds on similar findings in the prior research on price discrimination explanations of bundling. In a seminal piece, Stigler (1963) argued that bundling enables firms to extract more consumer surplus than unbundled sales when consumer preferences are negatively correlated. Other researchers have extended Stigler's findings to demonstrate that pure bundling is more profitable than unbundled sales as long as consumers' preferences are not perfectly positively correlated (e.g. Schmalensee, 1984; and McAfee *et al.*, 1989).

However, the explanation for bundling need not rely solely on price discrimination under imperfect competition. In a recent paper, Evans and Salinger (2005) insightfully point out that bundling is ubiquitous in competitive markets and provide numerous examples. They demonstrate that product-specific scale economies are a key factor in making bundling efficient. By limiting product selection through pure bundling, firms can reduce overall costs. This efficiency can exist regardless of whether firms have market power.

In sections 3 and 4 of our paper, we extend our analysis from a setting of monopoly to that of a duopoly by allowing entry. We examine the optimal prices and profits for the incumbent and the potential entrant. After some preliminary results, we set out a simple two-period game: in the first period, the incumbent chooses whether or not to bundle; in the second period, the entrant observes the incumbent's first period choices and decides whether or not to enter. If entry occurs then the optimal prices are determined by a Nash equilibrium under price competition. Equilibrium in this game is therefore defined as a pair of strategies that constitutes a Nash equilibrium in the pricing sub-game as well as the full game. While section 3 assumes entry can occur costlessly, we relax this assumption in section 4.

A comparison of the present value of the incumbent's two-period profits under different scenarios (i.e., combinations that arise from bundled versus unbundled sales and entry versus no entry) allows us to identify the optimal strategy for the incumbent. In contrast to the results of some previous studies, we show that pure bundling is not entry-deterring, but in fact makes profitable entry possible. Rather, an unbundled sale is the optimal strategy for the incumbent if entry costs are low because unbundled sales deter entry. Although the incumbent's first-period profits are lower with unbundled relative to bundled sales, the incumbent chooses not to bundle because bundling would allow profitable entry in the second period, eroding the incumbent's profits. Under plausible assumptions, the effect of entry-induced profit erosion in period 2 more than offsets the higher profits the incumbent would have earned from bundled sales in period 1. However, bundling becomes the optimal strategy when the entrant's cost of entry is high and the incumbent's monopoly position is not threatened. In this case, entry is deterred by high entry costs, not by bundling.

The finding that bundling is optimal when entry costs are high is consistent with Nalebuff's (2004) result. However, our conclusions regarding the implications of bundled sales on entry are different from those of Nalebuff because we consider the incumbent's optimal choices in a two-period framework, in which both the potential entrant and the incumbent choose optimal prices in a Nash setting.

Our findings are also markedly different from those of Whinston (1990). Like us, he considers a two-good setting, where the incumbent is a monopolist for the first good (good 1) and the potential entrant can only enter in the other good (good 2). Unlike us, however, in his initial set up, Whinston assumes that good 2 is differentiated but all customers have identical valuations for good 1. Consequently, if the entrant were to get any good 2 customers the incumbent, selling a bundle, would lose *all* of its good 1 customers. This provides a strong incentive for the incumbent to price the bundle so low as to make entry in good 2 unprofitable. The incumbent in Whinston's set up essentially uses its monopoly position in good 1 to cross-subsidize the good 2 in the bundle thereby denying the entrant any sales. Thus, in Whinston's model, the incumbent has to make a pre-commitment to bundling because, if entry occurs, the incumbent would prefer not to bundle[3]. By contrast, such pre-commitment is not necessary in our analysis, because, when faced with a potential entrant with low entry costs, the unbundling strategy produces the highest present value of profits for the incumbent.

Our analysis builds on related prior research by Carbajo *et al.* (1990) and Chen (1997). Carbajo *et al*. show that bundling may be profitable because it causes the rivals to act less aggressively; indeed, profitable bundling can raise the rival's payoffs if, absent bundling, competition is 'too' aggressive. Chen similarly shows that in a duopoly market, when at least one firm chooses to bundle, both firms earn positive profits even though they produce a homogenous good in that market and compete in prices. These results provide the underpinning and the economic intuition for our key finding: if the monopolist does not bundle, then prices for that good in the duopoly setting are driven down to marginal costs, making entry unprofitable for the rival (assuming infinitesimally small entry costs). By contrast, assuming entry costs are not prohibitive, bundling allows profitable entry because both firms, the incumbent and the entrant, earn positive profits for the good in which entry occurs.

2. The Firm's Decisions under Monopoly

We assume the following:

(1) A firm can adopt two alternative strategies: selling two independent goods, 1 and 2, at prices p_1 and p_2, or selling a 1-2 bundle at a price of P;
(2) Consumers are distributed in the reservation-price (or 'willingness-to-pay') space with unit density, and each consumer consumes a single unit of either or both goods;
(3) Consumers' reservation prices for each of the two goods, R_i (i = 1, 2), are distributed uniformly in the $\left[0, \overline{R}_i\right]$ interval, with $\overline{R}_i > 0$ for i = 1, 2. Thus, consumers with reservation prices greater than or equal to p_i will buy good i. For the sake of simplicity we assume that $\overline{R}_1 / \overline{R}_2 = r$, $\overline{R}_2 = 1$ and $r \geq 1$.

Given these assumptions, r represents the difference in consumers' maximum willingness to pay for good 1 relative to good 2. Importantly, r also denotes the size of market for good 1 relative to good 2, because a higher value of r indicates consumer preferences are more heterogeneous. Carbajo *et al.* (1990) as well as Nalebuff (2004) consider a special case in which r = 1. We demonstrate below that allowing r, which reflects the degree of heterogeneity of consumers' preferences for the two goods, to be different from unity significantly adds to the generality of our results.

Figure 1 depicts the demand for goods 1 and 2 without bundling. The rectangular area to the right of p_1 represents the demand for good 1, $D_1 = r - p_1$, while the rectangular area above p_2 represents the demand for good 2, $D_2 = r\,(1 - p_2)$.

Under bundling, consumers either buy or do not buy the bundled goods. Figure 2 depicts the demand for the bundled goods. There are three functional forms of demand depending on the values of P and r:

$$\text{Case A: } (P < r, P < 1) \qquad D^A = r - \frac{P^2}{2}$$

$$\text{Case B: } (1 \leq P < r) \qquad D^B = r - P + \frac{1}{2}$$

$$\text{Case C: } (1 \leq r < P) \qquad D^C = \frac{(1 + r - P)^2}{2}.$$

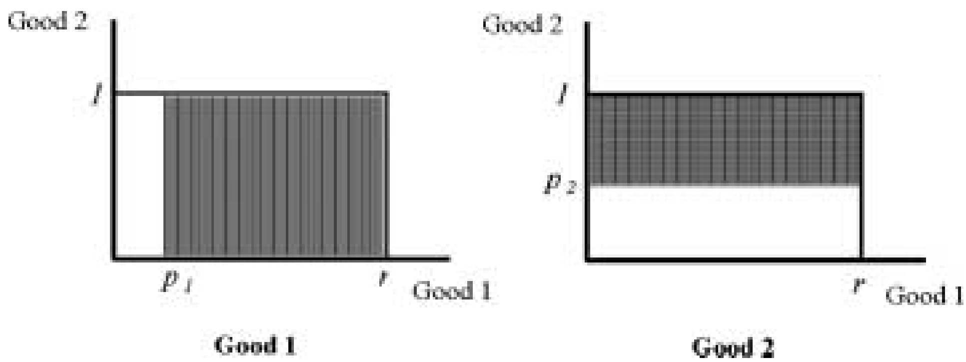

Figure 1. Demands for the unbundled goods under monopoly.

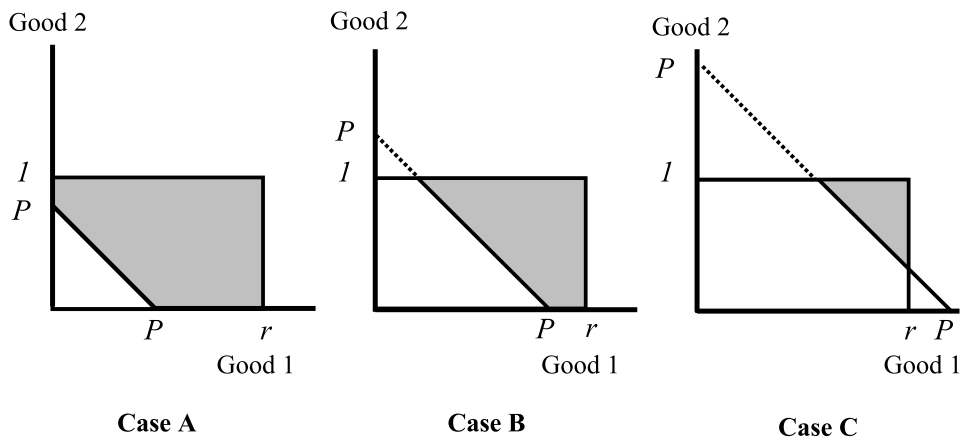

Figure 2. Demands for the bundled goods under monopoly.

It is obvious from above that if r were restricted to be unity then only Cases A and C would be valid, Case B would be ruled out. As we show below, Case C is never optimal for the monopolist; thus restricting $r = 1$, effectively constrains the monopolist's problem to Case A.

In all cases, consumers buy the bundle if the sum of their willingness to pay for goods 1 and 2 exceeds the price of the bundle; that is, if $R_1 + R_2 > P$. This inequality is represented by the area to the northeast of the 45° line in each reservation-price space. Figure 3 shows the demand for the bundled goods for all values of P. The figure illustrates that the demand function can be split into three sections: (A) $0 \leq P < 1$, (B) $1 \leq P < r$, and (C) $r \leq P < r + 1$. (Note: when $P = r + 1$, demand is equal to 0, as is clear from Case C in Figure 2.) At the juncture point P

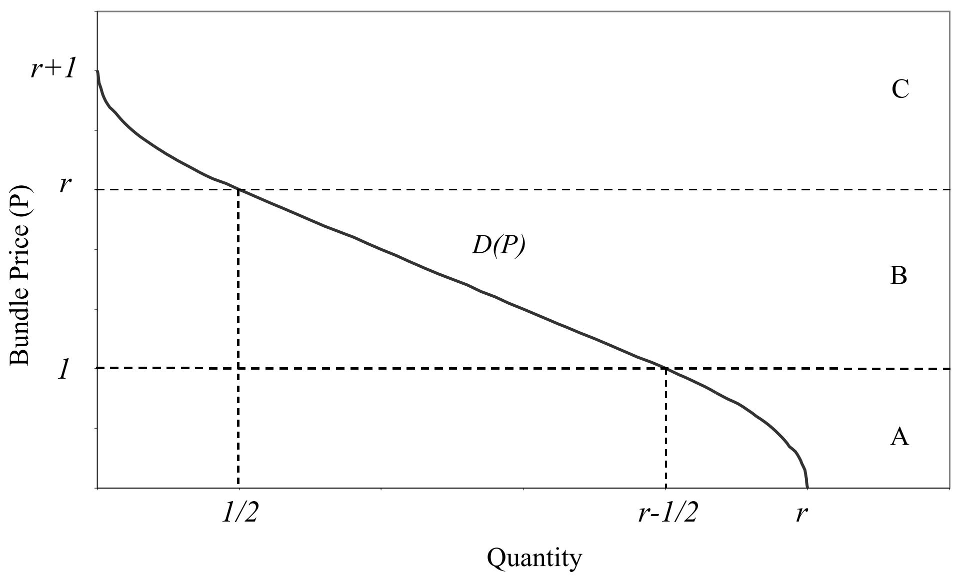

Figure 3. Demand function for the bundled goods.

$= 1$, $D^A = D^B$ and $\dfrac{dD^A}{dP} = \dfrac{dD^B}{dP}$. Similarly, at the other juncture point $P = r$, $D^B = D^C$ and $\dfrac{dD^B}{dP} = \dfrac{dD^C}{dP}$. Thus, the demand for the bundled goods is a well-defined and continuous function of P.

2.1 The Firm's Choice Problems

We assume throughout that the marginal costs for both goods are fixed and equal to zero (i.e., $c_1 = c_2 = 0$), which is likely for many computer-technology related products and information goods. Under unbundled pricing, the firm will choose optimal prices for good 1 (p_1^*) and good 2 (p_2^*) such that the profits are maximized in their respective markets. Specifically, the firm solves

$$\underset{p_1}{Max}\, \pi^1 = p_1(r - p_1)$$

$$\underset{p_2}{Max}\, \pi^2 = p_2(1 - p_2)r\cdot$$

Solutions to these problems provide the optimal price and the maximum profits in good 1 and good 2 as $p_1^* = r/2$, $\pi(p_1^*) = r^2/4$ and $p_2^* = 1/2$, $\pi(p_2^*) = r/4$, respectively. The second-order sufficient condition is satisfied for each good at the optimal price, which is unique and strictly positive.

We denote the sum of the optimal unbundled prices as $\overline{p}^* = p_1^* + p_2^* = (1 + r)/2$, and the monopolist's total profits at these prices are $\pi(p_1^*) + \pi(p_2^*) = (r^2 + r)/4$.

The firm's choice problem under bundling must be considered separately for each of the three cases shown in Figure 2. For example, in Case A, the firm's choice problem is $\underset{P}{Max}\, \pi^A = P(r - P^2/2)$. The optimal price under each case when substituted into the profit function yields the optimal profit level for each case.

Lemma 1 *The monopolist will never choose an optimal $P > r$, which occurs under Case C.*[4]

For the remainder of this paper, we will thus consider only Cases A and B. A comparison of the optimal prices and profits under these two cases shows that both optimal prices and profits for these cases are identical when $r = \frac{3}{2}$; they vary, however, for other values of r. Let the optimal bundled price be denoted by P^* and the maximum profits be denoted by $\pi(P^*)$.

Lemma 2 *If* $1 \le r < \frac{3}{2}$, *then* $P^* = \left(\frac{2r}{3}\right)^{\frac{1}{2}}$ *and* $\pi\left(P^*\right) = \left(\frac{2r}{3}\right)^{\frac{3}{2}}$ (*Case A*)

If $r \ge \frac{3}{2}$, *then* $P^* = \frac{2r+1}{4}$ *and* $\pi\left(P^*\right) = \frac{4r^2 + 4r + 1}{16}$ (*Case B*)

We now examine prices under bundled relative to unbundled sales.

Proposition 1: $P^* < (p_1^* + p_2^*) \equiv \overline{p}^*$ *for all values of r.*

Proposition 1 indicates that the bundled price is lower than the sum of the two unbundled prices. That is, every consumer who chooses to buy the bundle is better off buying the bundle at the bundled price rather than buying the two goods separately.

Note that both P^* and $\bar{p}^*$ are increasing functions of the degree of heterogeneity of consumers' preferences for the two goods, which we denote by r. The percentage difference between and P^* and $\bar{p}^*$, that is, $(P^* - \bar{p}^*)/\bar{p}^*$, exhibits a non-monotonous relationship with r. This difference increases for $1 \leq r < 1.5$, and falls for all $r \geq 1.5$. In the limit, the percent difference between P^* and $\bar{p}^*$ goes to zero.

Let us denote the difference between bundled and unbundled profits (evaluated at the respective optimal prices) as $\Delta_\pi \equiv \pi(P^*) - (\pi(p_1^*) + \pi(p_2^*))$. Let the percentage difference in the profits be denoted as $\Delta_\pi \% \equiv \Delta_\pi/(\pi(p_1^*) + \pi(p_2^*))$.

Proposition 2: $\Delta_\pi > 0$ *for all values of* r.

Proposition 2 shows that the monopolist always benefits from pure bundling relative to unbundled sales. Consumers' demand – and hence the monopolist's profits – under bundled and unbundled sales are increasing functions of r. Although the absolute difference in profits between bundled and unbundled sales increases with r, the percentage difference in profits, $\Delta_\pi\%$, decreases monotonically with r; it is highest when $r = 1$ and approaches zero as r goes to infinity. The economic intuition for this result is that, for very large values of r, consumers' preferences for the goods are so dissimilar that the *relative* gain from bundling disappears.

3. Duopoly

This section introduces an entrant to the monopoly market and examines the effect of entry on the incumbent's choices. We model a two-period game: In the first period, the incumbent chooses whether or not to bundle; in the second period, the entrant observes the incumbent's first period choice and decides whether or not to enter. We analyze prices and profits for the entrant and the incumbent under the settings of unbundled versus bundled sales. A comparison of the incumbent's profits under different scenarios allows us to identify the optimal strategy for the incumbent.

In addition to the demand and costs assumptions made in the preceding section, we add the following for the analysis of entry:

(1) The entrant can enter in only one of the goods 1 and 2;
(2) There are no barriers to entry. That is, the fixed cost of entry is zero[5].

3.1 Unbundled Sales

Under the assumption of Bertrand competition, entry to sell a good, whether it is good 1 or good 2, will cause the Bertrand-Nash equilibrium price to equal marginal cost. As a result, both the entrant and the incumbent will earn zero profits for the good in which entry occurs. For the other good the incumbent's profits will equal the monopoly profits for that good under unbundled sales, which we examined in the preceding section.

3.2 Bundling

An analysis of profits for the incumbent and the entrant for entry in goods 1 and 2 under bundling leads to the following lemma:

Lemma 3 *The incumbent's profits are highest when it bundles and no entry occurs. If entry is unavoidable, bundling and allowing entry in good 2 as opposed to good 1 are optimal for the incumbent. Also, the entrant's profits are higher if it enters in the market for good 2 rather than good 1.*

As a consequence of Lemma 3, in the remainder of this paper we will consider the economic implications of entry only in good 2. Thus, when we talk of entry it will mean entry in good 2. Additionally, before we can discuss the two-period game, we need to establish some preliminary results.

Figure 4 shows the demand for the entrant's good (D^e) and for the incumbent's bundle (D^I) within the reservation-price space when entry occurs under bundling[6]. The incumbent's bundled price and the entrant's price are denoted P and p^e, respectively. The price of the non-entry good implied by the bundled price is denoted as $\tilde{p} \equiv P - p^e$. For both Cases A and B, consumers buy the entrant's good if their willingness to pay for the good is higher than the entrant's price (i.e., $R_2 > p^e$) and if their willingness to pay for the other good is less than the price implied by the bundled price (i.e., $R_1 < \tilde{p}$). Thus, the demand for the entrant's good is the area above p^e and to the left of $\tilde{p}$ in the reservation-price space. Similarly, consumers will purchase the bundle if their willingness to pay for the non-entry good is larger than its implied price (i.e., $R_1 > \tilde{p}$) and if the sum of their willingness to pay for both goods is larger than the bundled price (i.e., $R_1 + R_2 > P$). These two conditions are satisfied in the area to the northeast of the 45° line and to the right of $\tilde{p}$.

Both Cases A and B yield identical demand functions. Specifically, the demand functions are $D^e = (P - p^e)(1 - p^e)$ for the entrant and $D^I = (r - P + p^e) - (p^e)^2/2$ for

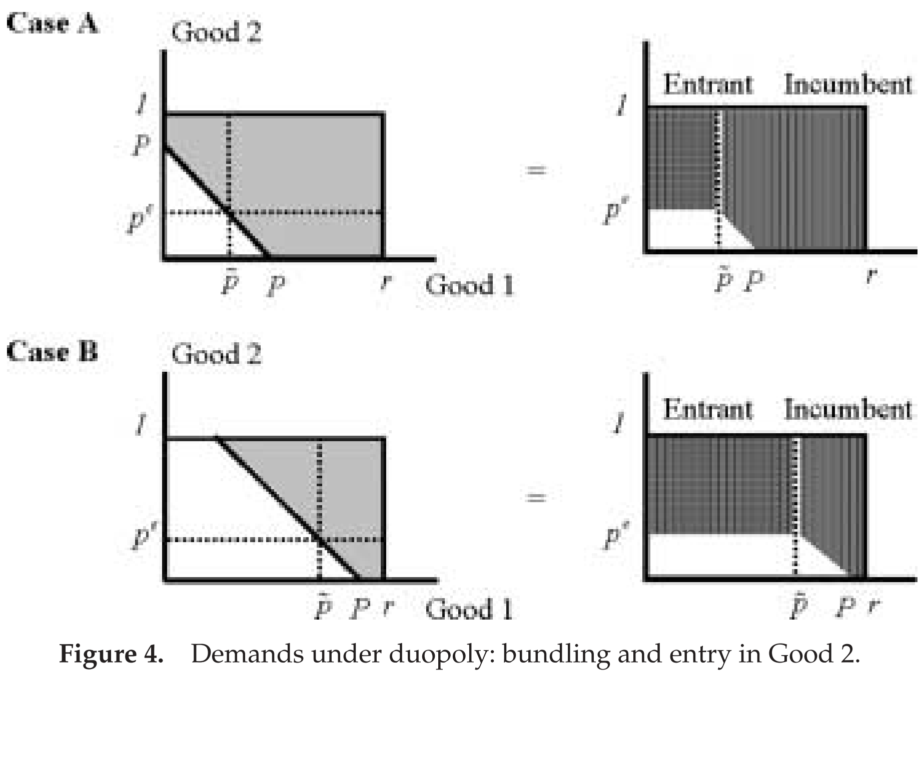

Figure 4. Demands under duopoly: bundling and entry in Good 2.

the incumbent. Importantly, the entrant's demand is independent of r; it is determined purely by its and the incumbent's prices.

The optimal prices, P^* and p^{e^*}, will form the Nash equilibrium that simultaneously satisfies the two first-order conditions (FOCs), one for the entrant and the other for the incumbent:

$$\frac{\partial \pi^e}{\partial p^e} = \frac{\partial\left\{D^e\left(P,p^e\right)\cdot p^e\right\}}{\partial p^e} = 3\left(p^e\right)^2 + P - 2p^e(1+P) = 0$$

$$\frac{\partial \pi^I}{\partial P} = \frac{\partial\left\{D^I\left(P,p^e\right)\cdot P\right\}}{\partial P} = p^e - \frac{\left(p^e\right)^2}{2} + r - 2P = 0.$$

There are no closed-form solutions to these FOCs; as a result, we use numerical solutions. Solutions of these FOCs that satisfy the constraints $0 < p^e < P$ and $0 < P < r$, and also satisfy the second-order sufficient conditions

$$\frac{\partial^2 \pi^e}{\partial\left(p^e\right)^2} = 6p^e - 2(1+P) < 0 \text{ and } \frac{\partial^2 \pi^I}{\partial P^2} = -2 < 0,$$

are the Nash equilibrium prices.

The fact that the Nash equilibrium prices are well-defined and strictly interior becomes clear by examining the reaction functions of the entrant and the incumbent. The reaction functions are derived from the FOCs noted above. Specifically, they are $P(p^e) = (3(p^e)^2 - 2p^e)/(1 - 2p^e)$ for the entrant and $P(p^e) = (2p^e-(p^e)^2 + 2r)/4$ for the incumbent (both expressed as a function of p^e for the purposes of graphical illustration). Figure 5 shows these functions for several values of r. For any given r, the Nash equilibrium prices, P^* and p^{e^*}, are determined at the intersection of the two functions.

Figure 5 shows, as r increases, the incumbent's optimal price, P^*, rapidly increases, while the entrant's optimal price, p^{e^*}, remains relatively unchanged. This finding accords with intuition because, with entry, the incumbent still holds a monopoly over the other good (i.e., good 1). The relatively more valuable good 1 is to consumers (i.e., the higher the value of r), the greater the incumbent's ability to charge higher prices. By contrast, the entrant's optimal price is not directly affected by r because the entrant's demand function is independent of r; rather, it is indirectly affected by changes in the incumbent's optimal bundled price, which varies with r.

3.3 The Incumbent's Choices in Two Periods

We can now model the incumbent's choices in a two-period setting. In the first period, the incumbent decides whether or not to bundle. Based on this choice, in the second period, the entrant decides whether or not to enter. Figure 6 illustrates the set-up of the two-period game.

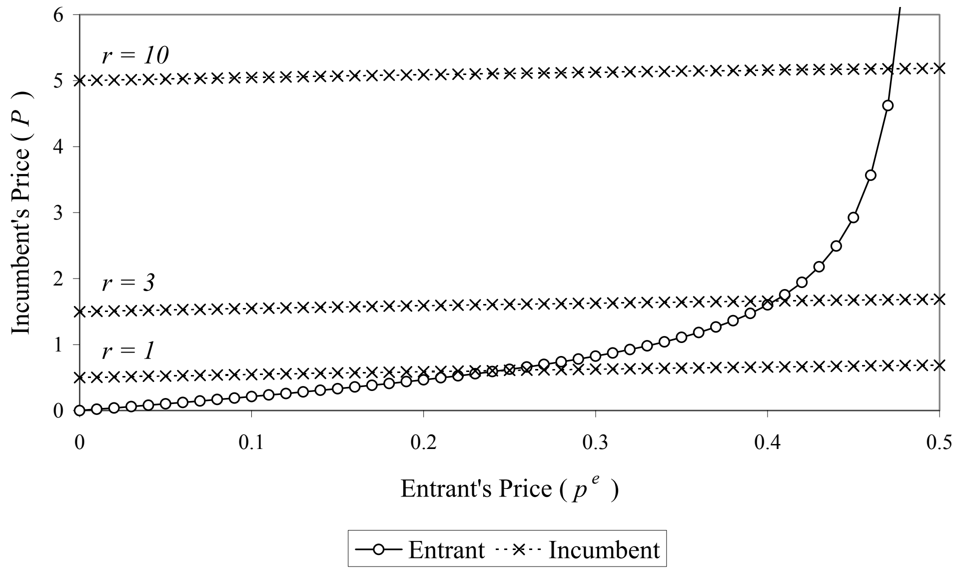

Figure 5. Reaction functions under bundling and entry in Good 2.

We posit that the incumbent will calculate the present value of profits from the two periods (π^I) for the four different paths and choose the strategy that yields the highest present value of profits:

$$\pi^I = \pi^I_{s_1} + \frac{\pi^I_{s_2}}{(1+\rho)}$$

$\pi^I_{s_i}$ = incumbent's profit under scenario s in period i (i = 1, 2)
s = scenario (i.e., [Bundle, No Entry], [Bundle, Entry], [No Bundle, No Entry], [No Bundle, Entry])
ρ = discount rate.

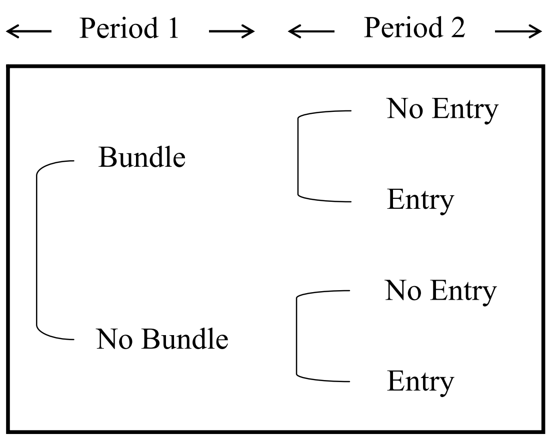

Figure 6. Incumbent's two-period game.

In Figure 7, we illustrate the incumbent's profits for the different scenarios under the assumption that r = 2 and ρ = 0.05. Our numerical analyses show that for all values of r, the scenario [Bundle, No Entry] would produce the largest profits for the incumbent. The scenario is unrealistic, however, because, with zero entry costs, the entrant makes profits under bundling and would therefore enter. Thus, [Bundle, No Entry] is not a viable scenario when there is no cost of entry. If the incumbent does not bundle in the first period, the entrant makes no profits under Bertrand competition and entry is deterred as a result[7]. This outcome suggests that [No Bundle, Entry] is not a realistic scenario, either.

Thus, to the incumbent, the relevant choices are: bundle and allow entry in the second period (i.e., [Bundle, Entry]) or not bundle and enjoy monopoly profits in both periods (i.e., [No Bundle, No Entry]). In the second choice, the incumbent's profits are lower in the first period because profits are lower with unbundled sales than they are with bundled sales, but profits are higher in the second period because monopoly profits with unbundled sales are higher than Nash profits under duopoly. Numerical analysis shows that for all reasonable values of the discount rate the second of the two choices (i.e., [No Bundle, No Entry]) dominates. This result reverses only if the discount rate increases to an unrealistic and extraordinary level. For example, for r = 2, [Bundle, Entry] is optimal only when the discount rate reaches 200%.

Proposition 3: *In a two-period set up, the incumbent maximizes the present value of its profits by choosing unbundled sales because that choice deters entry.*

The foregoing analysis suggests that, contrary to the results in the existing literature, it is bundling by the incumbent that makes profitable entry possible. To deter entry and maximize the present value of two-period profits, the incumbent chooses unbundled sales. The economic intuition for this key finding is as follows: if the monopolist does not bundle, then prices in the duopoly setting for that good are driven down to marginal costs, making entry unprofitable for the rival (assuming infinitesimally small entry costs). By contrast, bundling allows

Scenario		Profit		
Period 1	Period 2	Period 1	Period 2	Present Value
Bundle	No Entry	1.56	1.56	**3.05**
	Entry	1.56	1.31	**2.81**
No Bundle	No Entry	1.50	1.50	**2.93**
	Entry	1.50	1.00	**2.45**

Figure 7. Incumbent's two-period game: an example (r = 2 and ρ = 0.05).

profitable entry because both firms, the incumbent and the entrant, earn positive profits for the good for which entry occurs.

4. The Incumbent's Choices with Positive Fixed Cost of Entry

We have so far assumed that the entrant incurs no fixed costs or entry costs. As a consequence, bundling by a monopolist induces entry because the entrant always makes profits under this assumption. We now relax this assumption and suppose that the entrant incurs a fixed cost denoted by k. With positive fixed costs, bundling does not necessarily induce entry because the entrant's profits (net of fixed costs) at the Nash equilibrium prices may be zero or negative. Therefore, the [Bundle, No Entry] path in the two-period set-up becomes possible when $k > 0$. For example, when $r = 2$, the entrant's profit without fixed costs at Nash equilibrium prices (π^{e^*}) equals 0.18; thus, entry is deterred despite bundling by the incumbent if $k > 0.18$. However, even in the case where the entrant's entry costs (or fixed costs) are low – in this example, say $k < 0.18$ – the incumbent may choose a bundled price such that the entrant's profit[8] net of entry costs is negative, making entry unprofitable.

A similar analysis is at the heart of Nalebuff's (2004) arguments about the entry-deterring effects of bundling. We show below that such a strategy is suboptimal (i.e., yields lower present value of two-period profits) for the incumbent relative to the strategy of unbundled sales. The fact that such a strategy is suboptimal is also recognized by Whinston (1990). He writes: 'Note that if both firms are active, firm 1's [i.e., the incumbent's] profits are also *lower* in the bundling regime than under independent pricing.…Thus, in this model, firm 1 would never commit to tying unless this would succeed in driving firm 2 [i.e., the entrant] out of the market.' [p. 844, emphasis added] Thus, Whinston's argument is akin to one of predatory pricing: the incumbent pre-commits to bundling despite lower profits[9] so that it can convert the market in which potential entry occurs from a duopoly to a monopoly. We demonstrate below that such a pricing strategy can be extremely costly (in terms of foregone profits) to the incumbent.

4.1 Strategic Pricing

Even in cases in which the entrant's fixed costs are low (for example, for $r = 2$ when $k < 0.18$), the incumbent may deter entry through 'strategic pricing,' that is, by choosing a price that would constrain the entrant's profits net of fixed costs to be zero. The incumbent's choice problem under this pricing strategy can be formulated as $\underset{P}{Max}\,\pi^A = D^I(P, p^e) \cdot P$ subject to $D^e(P, p^e) \cdot p^e - k = 0$,
where $D^I(P, p^e)$ denotes the incumbent's demand function under bundling and $D^e(P, p^e)$ denotes the entrant's demand.

We denote the incumbent's price choice under strategic pricing by $P^{**}(k)$. Substituting $P^{**}(k)$ in the incumbent's profit function yields the profits from strategic pricing $\pi^{I^{**}}(k)$. One can readily verify that, for all values of r, both $P^{**}(k)$ and $\pi^{I^{**}}(k)$ are increasing functions of the entrant's fixed cost k.

Because the incumbent's choice problem under strategic pricing is a constrained version of the one under Nash equilibrium, optimal profits are lower. Indeed, numerical analysis shows that the incumbent's profits under strategic pricing ($\pi^{I^{**}}$) are always suboptimal to profits under Nash equilibrium (π^{I^*}). That is, $\pi^{I^{**}} < \pi^{I^*}$ for $0 \le k < \pi^{e^*}$, for all $r \ge 1$. Strategic pricing becomes irrelevant for $k > \pi^{e^*}$ because the entrant's profits under Nash prices no longer cover its fixed costs, and thus entry does not occur even when the incumbent bundles.

4.2 A Synthesis

Figure 8 synthesizes the analyses of the incumbent's profits under different pricing schemes. As noted above, in the region in which $k < \pi^{e^*}$, strategic pricing ([Bundle, No Entry]) is suboptimal, and the incumbent's profits are lower than they would be if entry were allowed through bundling and Nash equilibrium prevailed ([Bundle, Entry]). However, the incumbent can make higher profits by deterring entry through unbundling ([No Bundle, No Entry]). Thus, deterring entry through strategic pricing under bundling is the least profitable among the three possible choices. By adopting strategic pricing, the incumbent gives up profits of Δ_1 relative to the optimal choice of unbundled monopoly ([No Bundle, No Entry]). Figure 8 also shows that the value of Δ_1 increases as k decreases. This result is intuitive; when the cost of entry is low, entry is easier and, as a result, strategic pricing is costlier to the incumbent in terms of profits sacrificed. The numerical analysis in Figure 8 shows that in percentage terms the cost of adopting strategic pricing can be as high as 50 percent when $r = 2$.

In Figure 8, in the region in which $k > \pi^{e^*}$, entry will not occur even when the incumbent bundles, because the entrant's profits under Nash equilibrium net of fixed costs will be negative. Because there is no threat of entry, the incumbent realizes the greatest profits by bundling and remaining a monopolist ([Bundle, No Entry]) – a possibility that was ruled out when the entrant's fixed costs were

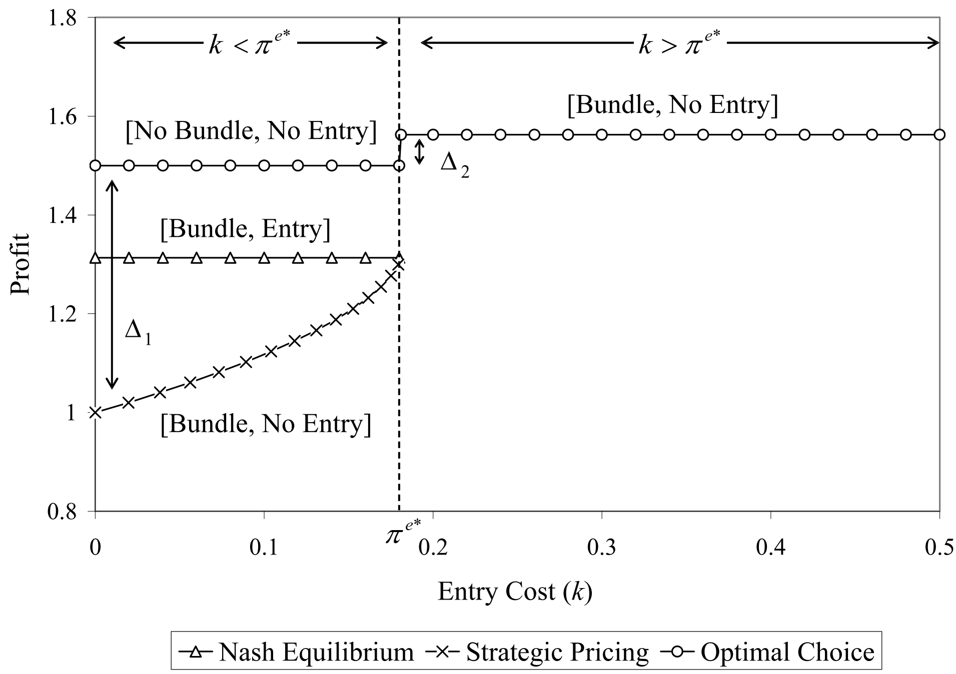

Figure 8. Incumbent's profit under various pricing schemes ($r = 2$).

assumed to be zero. The symbol Δ_2 denotes the incremental profits resulting from bundling relative to unbundled sales. In this case, entry is deterred by the entrant's high fixed costs, not by the incumbent's bundled sales.

The foregoing results can be summarized as follows:

Proposition 4: *If $k < \pi^{e^*}$, the optimal choice for the incumbent is to sell the two goods independently and deter entry; if $k \geq \pi^{e^*}$, the optimal choice for the incumbent is to sell the goods as a bundle.*

In our analyses, we have assumed a time lag between the incumbent's choice of whether or not to bundle and the entrant's entry decision. We also explore whether our results would change if the incumbent could instantaneously switch between bundled and unbundled sales. For example, it is conceivable that the incumbent would choose to bundle while there is no possibility of entry on the horizon, but would switch instantly to unbundled sales to force the entrant to exit if entry occurred. Alternatively, the incumbent could threaten to unbundle if entry were to occur and, if successful, it would be able to continue bundling and deter entry even when the entrant's entry cost was low. In these examples, the incumbent deters entry through the act or threat of unbundling, not bundling. Consequently, the assumption of an instantaneous switch between bundled and unbundled sales does not change the fundamental results of the paper.

4.3 Potential Extensions of Our Research

Our analysis can be extended on several fronts. First, the distribution of consumer's preferences can be generalized from the Uniform distribution assumed in the paper to a distribution without a specific functional form. However, given the absence of closed form solutions even under the simplifying assumption of a Uniform distribution, it is likely one will have to posit a specific functional form to derive qualitative results. For example, following Schmalensee (1984), one could assume that consumer preferences are normally distributed and derive results through numerical analysis. Second, one could investigate whether the key results of the paper hold if demand complementarity between the two goods is admitted. Modeling this can be relatively straightforward under normality, since one can assume a bi-variate normal distribution for consumers' preferences for the two goods, with the degree of complementarity reflected through the coefficient of correlation. Finally, in light of Evans and Salinger (2005), it may be informative to explore how our results are impacted if bundling-induced efficiencies are introduced.

5. Conclusion

We have shown that under the setting of monopoly, the optimal price of a bundle of goods with independent demands is lower, while profits are higher, than prices and profits under unbundled sales. Therefore, the monopolist's optimal strategy is to bundle.

We have also shown that, in contrast to the results of some previous studies, pure bundling is not entry-deterring in a potential duopoly with low entry costs, but in fact makes profitable entry possible. Unbundled sales, which deter entry, are thus the optimal strategy for the incumbent. Bundling becomes the optimal strategy when the entrant's cost of entry is high and the incumbent's monopoly position is not threatened. In this case, entry is deterred by high entry costs and not by bundling.

Notes

1. A referee points out although that is true in the US, European antitrust agencies have expressed concerns about mixed bundling.
2. By independent demand we mean that consumers' preferences for the two goods are uncorrelated; that is, the value derived from one good is not enhanced by the consumption of the other good. Consequently, we do not assume demand complementarity.
3. See Nalebuff (2004) for an excellent critique of Whinston (1990).
4. We present the proofs of the lemmas and propositions in the Appendix.
5. We relax assumption (2) later in this paper.
6. We consider only Cases A and B because it can be readily verified that Case C (i.e., $P > 1$ and $P > r$) is never optimal for the incumbent.
7. Strictly speaking the entrant should be indifferent between entry and no entry since in both cases profits are zero; the no-entry outcome can be reasoned by assuming infinitesimally small entry costs.
8. Recall from the discussion in the previous section that the entrant's profit is a function of not only its price but also of the bundled prices charged by the incumbent.
9. Indeed, as Nalebuff points out, it is precisely because of this reason pre-commitment to bundling is necessary in Whinston's model.

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Appendix. Proofs of Lemmas and Propositions.

Lemma 1 *The monopolist will never choose an optimal P > r, which occurs under Case C.*

Proof: First, we find the solution to the firm's problem for each of the Cases A, B, and C.

Case A: $$\underset{P}{Max}\ \pi^A = P\left(r - \frac{P^2}{2}\right) \tag{1}$$

FOC: $\frac{d\pi^A}{dP} = r - \frac{3}{2}P^2 = 0$, SOC: $\frac{d^2\pi^A}{dP^2} = -3P < 0$

$$\therefore P_A^* = \left(\frac{2}{3}r\right)^{\frac{1}{2}} \text{ for } 1 \le r < \frac{3}{2} \tag{2}$$

Note that P_A^* satisfies the Case A constraint $P < r$, $P < 1$ for $1 \le r < \frac{3}{2}$.

Case B: $$\underset{P}{Max}\ \pi^B = P\left(r - P + \frac{1}{2}\right) \tag{3}$$

FOC: $\frac{d\pi^B}{dP} = r - 2P + \frac{1}{2} = 0$, SOC: $\frac{d^2\pi^B}{dP^2} = -2 < 0$

$$\therefore P_B^* = \frac{2r+1}{4} \text{ for } r > \frac{3}{2} \tag{4}$$

Note that P_B^* satisfies the Case B constraint $1 < P < r$ for $r > \frac{3}{2}$.

Case C: $$\underset{P}{Max}\ \pi^C = \frac{P(1+r-P)^2}{2}$$

FOC: $\frac{d\pi^C}{dP} = \frac{(1+r-P)(1+r-3P)}{2} = 0$, SOC: $\frac{d^2\pi^C}{dP^2} = 3P - 2r - 2 < 0$

For $\pi^C > 0$, the solution to FOC implies $P_c = \frac{r+1}{3}$. It satisfies the Case C constraint $r < P$ only when $r < \frac{1}{2}$, which violates the assumption $r \ge 1$. Therefore, Case C is not feasible.

Lemma 2 *If* $1 \le r < \frac{3}{2}$, *then* $P^* = \left(\frac{2r}{3}\right)^{\frac{1}{2}}$ *and* $\pi\left(P^*\right) = \left(\frac{2r}{3}\right)^{\frac{3}{2}}$ *(Case A)*

If $r \ge \frac{3}{2}$, *then* $P^* = \frac{2r+1}{4}$ *and* $\pi\left(P^*\right) = \frac{4r^2+4r+1}{16}$ *(Case B)*

Proof: We derive the maximum levels of profits by substituting P_A^* from equation (2) for P in the profit function shown in equation (1) and by substituting P_B^* from equation (4) for P in the profit function shown in equation (3):

i) $1 \le r < \frac{3}{2}$: $P^* = P_A^* = \left(\frac{2}{3}r\right)^{\frac{1}{2}}$ $\therefore \pi\left(P^*\right) = \pi\left(P_A^*\right) = \left(\frac{2}{3}r\right)^{\frac{3}{2}}$

ii) $r \ge \frac{3}{2}$: $P^* = P_B^* = \frac{2r+1}{4}$ $\therefore \pi\left(P^*\right) = \pi\left(P_B^*\right) = \frac{4r^2+4r+1}{16}$

Proposition 1: $P^* < (p_1^* + p_2^*) \equiv \bar{p}^*$ *for all values of r.*

Proof:

i) $1 \le r < \frac{3}{2}$

By Lemma 2, $P^* = \left(\frac{2}{3}r\right)^{\frac{1}{2}}$, and from section 2, $\bar{p}^* = p_1^* + p_2^* = \frac{r+1}{2}$.

$$P^{*2} - \bar{p}^{*2} = \frac{2r}{3} - \frac{(r+1)^2}{4} = -\frac{1}{4}\left\{\left(r - \frac{1}{3}\right)^2 + \frac{8}{9}\right\} < 0$$

$\therefore$ $P^* < \bar{p}^*$ (since by definition, $P^* > 0$, $\bar{p}^* > 0$)

ii) $r \ge \frac{3}{2}$

By Lemma 2, $P^* = \frac{2r+1}{4}$, and from section 2, $\bar{p}^* = p_1^* + p_2^* = \frac{r+1}{2}$.

$$P^* - \bar{p}^* = \frac{2r+1}{4} - \frac{r+1}{2} = -\frac{1}{4} < 0$$

$\therefore$ $P^* < \bar{p}^*$

Proposition 2: $\Delta_\pi > 0$ *for all values of r.*

Proof:

i) $1 \le r < \frac{3}{2}$

By Lemma 2, $\pi\left(P^*\right) = \left(\frac{2}{3}r\right)^{\frac{3}{2}}$, and from section 2, $\pi(p_1^*) + \pi(p_2^*) = \frac{r^2+r}{4}$.

$$\Delta_\pi \equiv \pi\left(P^*\right) - \left(\pi\left(p_1^*\right) + \pi\left(p_2^*\right)\right) = \left(\frac{2}{3}r\right)^{\frac{3}{2}} - \frac{r^2+r}{4}$$

$$\Delta_\pi = \frac{1}{6} > 0 \text{ for } r = 1, \quad \frac{d\Delta_\pi}{dr} = \left(\frac{2}{3}r\right)^{\frac{1}{2}} - \frac{r}{2} - \frac{1}{4} > 0 \text{ for } 1 \le r < \frac{3}{2}$$

$\therefore$ $\Delta_\pi > 0$

ii) $r \ge \frac{3}{2}$

By Lemma 2, $\pi\left(P^*\right) = \frac{4r^2+4r+1}{16}$, and from section 2, $\pi(p_1^*) + \pi(p_2^*) = \frac{r^2+r}{4}$.

$$\Delta_\pi \equiv \pi\left(P^*\right) - \left(\pi\left(p_1^*\right) + \pi\left(p_2^*\right)\right) = \frac{4r^2 + 4r + 1}{16} - \frac{r^2 + r}{4} = \frac{1}{16} > 0$$

$$\therefore \ \Delta_\pi > 0$$

Extension of Proposition 2: Δ_π *and* $\Delta_\pi\%$ as functions of r.

Figures 9 and 10 show Δ_π and $\Delta_\pi\%$ as functions of r. As explained in section 2, $\Delta_\pi\% \equiv \Delta_\pi / \left(\pi\left(p_1^*\right) + \pi\left(p_2^*\right)\right)$. The profit difference, Δ_π, reaches its maximum at $r = \frac{3}{2}$ and then remains flat at $\frac{1}{16}$ for all $r > \frac{3}{2}$. The percentage profit difference, $\Delta_\pi\%$, is highest at $r = 1$, and then as r goes to infinity, $\Delta_\pi\%$ approaches zero.

Lemma 3 *The incumbent's profits are highest when it bundles and no entry occurs. If entry is unavoidable, bundling and allowing entry in good 2 as opposed to good 1 are optimal for the incumbent. Also, the entrant's profits are higher if it enters in the market for good 2 rather than good 1.*

Proof: Proof of Lemma 3 requires examination of the maximum profits of the entrant and the incumbent under each of the following combinations of the entrant's and the incumbent's decisions: [Bundle, No Entry], [Bundle, Entry], [No Bundle, No Entry], and [No Bundle, Entry]. Scenarios involving 'Entry' are subdivided into 'Entry in Good 1' and 'Entry in Good 2.' That is, [Bundle, Entry] is separated into [Bundle, Entry in Good 1] and [Bundle, Entry in Good 2], and [No Bundle, Entry] is separated into [No Bundle, Entry in Good 1] and [No Bundle, Entry in Good 2]; i) Analysis of Profits under Various Scenarios

Scenario: [Bundle, No Entry]

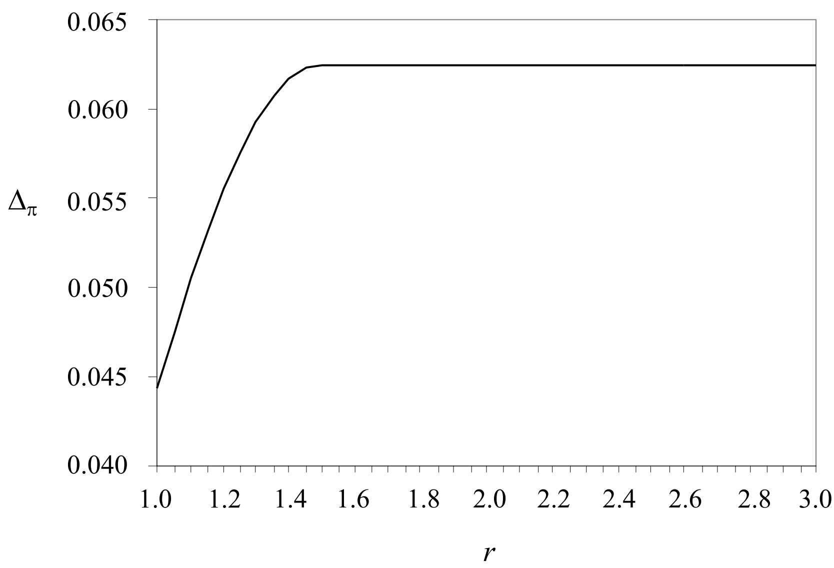

Figure 9. Δ_π as a function of r.

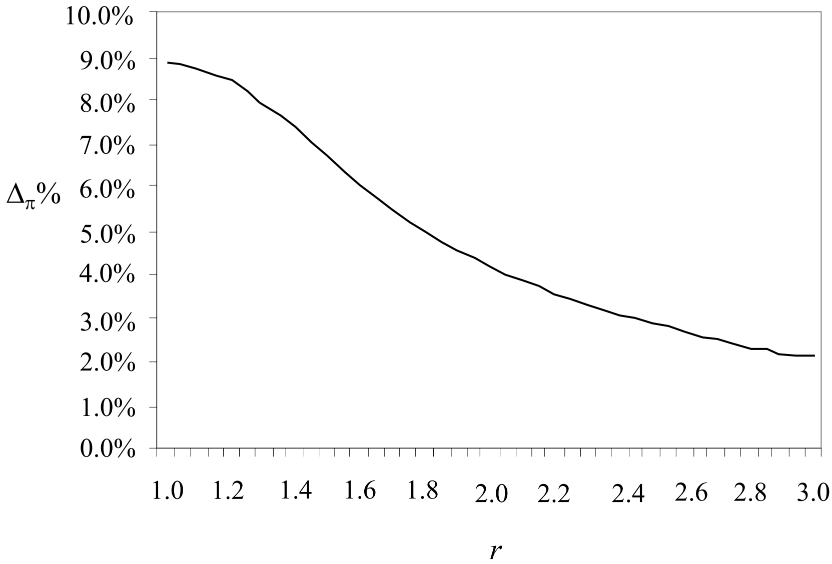

Figure 10. Δ_π% as a function of *r*.

This scenario is equivalent to 'bundling under monopoly.' Therefore, Lemma 2 gives the incumbent's profits. The entrant's profits are not relevant because entry does not occur:

$$\pi^{I*} = \left(\frac{2}{3}r\right)^{\frac{3}{2}} \text{ for } 1 \le r < \frac{3}{2} \text{ and } \pi^{I*} = \frac{4r^2 + 4r + 1}{16} \text{ for } r \ge \frac{3}{2}. \tag{5}$$

Scenario: [No Bundle, No Entry]

This scenario is equivalent to 'unbundled sales under monopoly.' We discuss the incumbent's profits in section 2 of the paper. The entrant's profits are not relevant because entry does not occur:

$$\pi^{I*} = \frac{r^2 + r}{4} \tag{6}$$

Scenario: [No Bundle, Entry in Good 1]

As we describe in section 3, entry in a good drives down the price of the entry good to its marginal cost under the assumption of Bertrand competition. The marginal costs for both goods 1 and 2 are assumed to be zero. Therefore, the case [No Bundle, Entry in Good 1] will be associated with zero profits from good 1 for both the entrant and the incumbent. Hence, the entrant's profits are zero. The incumbent, however, is still a monopolist on good 2, and, thus, the incumbent's profits equal the profits from independent sale of good 2 under monopoly, which we derived in section 2:

$$\pi^{I*} = \frac{r}{4}, \text{ and} \tag{7}$$

$$\pi^{e^*} = 0. \tag{8}$$

Scenario: [No Bundle, Entry in Good 2]

The same logic holds for [No Bundle, Entry in Good 2] as for [No Bundle, Entry in Good 1]. The entrant's profits are zero, and the incumbent's profits equal the profits from independent sale of good 1 under monopoly:

$$\pi^{I^*} = \frac{r^2}{4}, \text{ and} \tag{9}$$

$$\pi^{e^*} = 0. \tag{10}$$

Scenario: [Bundle, Entry in Good 1]

Figure 11 illustrates the demands for the entrant's good (D^e) and for the incumbent's bundle (D^I) within the reservation-price spaces for which entry occurs in good 1 under bundling.

We denote the incumbent's bundled price and the entrant's price by P and P^e, respectively. The price of good 2 implied by the bundled price is defined as $\tilde{p} \equiv P - p^e$. For both Cases A and B, consumers buy the entrant's good if their willingness to pay for good 1 is higher than the entrant's price and if their willingness to pay for good 2 is less than the price implied by the bundled price. It is the area to the right of p^e and below $\tilde{p}$ in the reservation-price space. Similarly, consumers demand the bundle if their willingness to pay for good 2 is larger than its implied price and if the sum of their willingness to pay for both goods is larger than the bundled price. It is the area to the northeast of the 45° line and above $\tilde{p}$. Not shown in Figure 11 is Case C in which $P > 1$ and $P > r$. There are no solutions that maximize profits and satisfy these constraints; thus, Case C is not feasible.

The mathematical expressions for demands are as follows:

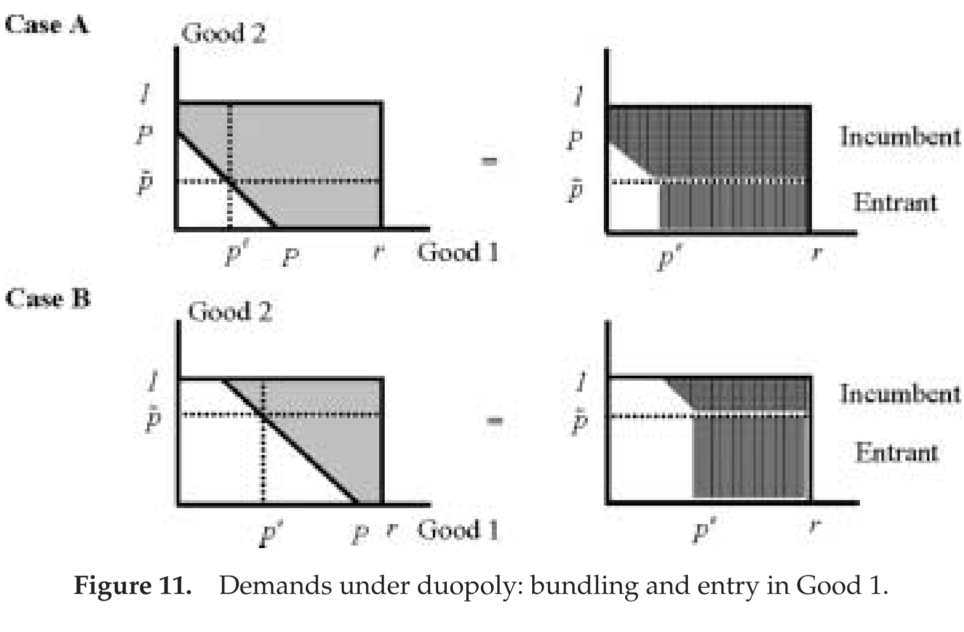

Figure 11. Demands under duopoly: bundling and entry in Good 1.

Case A: ($P < r$, $P < 1$)

$D^e = (P - p^e)\,(r - p^e)$, and

$$D^I = r\left(1 - P + p^e\right) - \frac{\left(p^e\right)^2}{2}.$$

Case B: ($1 < P < r$)

$D^e = (P - p^e)\,(r - p^e)$, and

$$D^I = \left(r - p^e\right)\left(1 - P + p^e\right) + \frac{\left(1 - P + p^e\right)^2}{2}.$$

To identify the Nash equilibrium prices, we must solve two first-order conditions – one for the entrant and the other for the incumbent – simultaneously for each of the Cases A and B. There are no closed-form solutions to their first-order conditions for the entrant and the incumbent. Therefore, we employ numerical methods to conduct analyses. The solutions are the set of optimal prices, p^* and p^{e*}, that form the Nash equilibrium.

Case A: ($P < r$, $P < 1$)

$$FOC^e: \frac{\partial \pi^e}{\partial p^e} = \frac{\partial\left\{D^e\left(P, p^e\right) \cdot p^e\right\}}{\partial p^e} = 3\left(p^e\right)^2 + rP - 2p^e(r + P) = 0, \text{ and} \tag{11}$$

$$FOC^I: \frac{\partial \pi^I}{\partial P} = \frac{\partial\left\{D^I\left(P, p^e\right) \cdot P\right\}}{\partial P} = -\frac{\left(p^e\right)^2}{2} + r + p^e r - 2rP = 0. \tag{12}$$

The solutions to these first-order conditions must also meet the second-order conditions:

$$SOC^e: \frac{\partial^2 \pi^e}{\partial \left(p^e\right)^2} = 6p^e - 2(r + P) < 0, \quad \text{and}$$

$$SOC^I: \frac{\partial^2 \pi^I}{\partial P^2} = -2r < 0.$$

Case B: ($1 < P < r$)

$$FOC^e: \frac{\partial \pi^e}{\partial p^e} = \frac{\partial\left\{D^e\left(P, p^e\right) \cdot p^e\right\}}{\partial p^e} = 3\left(p^e\right)^2 + rP - 2p^e(r + P) = 0, \text{ and}$$

$$FOC^I: \frac{\partial \pi^I}{\partial P} = \frac{\partial\left\{D^I\left(P, p^e\right) \cdot P\right\}}{\partial P} = \frac{1}{2} - \frac{\left(p^e\right)^2}{2} + r + p^e r - 2P - 2rP + \frac{3\left(p^e\right)^2}{2} = 0.$$

The corresponding second-order conditions are

$$SOC^e: \frac{\partial^2 \pi^e}{\partial \left(p^e\right)^2} = 6p^e - 2(r + P) < 0, \text{ and}$$

$$SOC^I : \frac{\partial^2 \pi^I}{\partial P^2} = -2 - 2r + 3P < 0.$$

However, there are no solutions for Case B that satisfy the required constraints. Therefore, Case B for entry in good 1 is not feasible. We consider only Case A for further analysis.

From the first-order conditions in equations (11) and (12), and given the constraints for Case A, we derive the reaction functions of the entrant and the incumbent, expressed as functions of p^e, as follows:

Entrant: $$P(p^e) = \frac{3(p^e)^2 - 2p^e r}{2p^e - r} \tag{13}$$

Incumbent: $$P(p^e) = \frac{2r + 2p^e r - (p^e)^2}{4r}. \tag{14}$$

Figure 12 illustrates these reaction functions (equations (13) and (14)) for $r = 2$. The Nash equilibrium and corresponding optimal prices, P^* and p^{e*}, are determined at the intersection of the two reaction functions.

Plugging P^* and p^{e*} into the following profit functions yields the maximum profits to the incumbent and the entrant for good 1 entry under bundling:

Entrant: $$\pi^e = D^e(P, p^e) \cdot p^e = p^e (r - p^e)(P - p^e) \tag{15}$$

Incumbent: $$\pi^I = D^I(P, p^e) \cdot P = P\left\{ r(1 + p^e - P) - \frac{(p^e)^2}{2} \right\}. \tag{16}$$

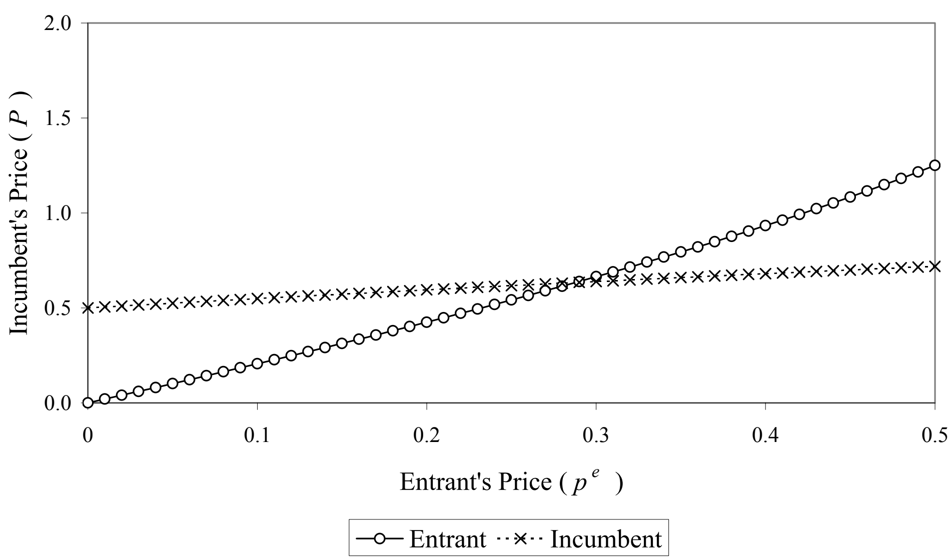

Figure 12. Reaction functions under bundling and entry in good 1 ($r = 2$).

Scenario: [Bundle, Entry in Good 2]

The same procedure as that in [Bundle, Entry in Good 1] applies and the analysis is discussed in section 3 of the paper. The reaction functions are:

Entrant: $$P\left(p^e\right)=\frac{3\left(p^e\right)^2-2p^e}{1-2p^e} \tag{17}$$

Incumbent: $$P\left(p^e\right)=\frac{1}{4}\left\{2p^e-\left(p^e\right)^2+2r\right\}. \tag{18}$$

We solve for P^* and p^{e*} at the point that equates equations (17) and (18), and we substitute the values into the profit functions below:

Entrant: $$\pi^e=D^e\left(P,p^e\right)\cdot p^e=p^e\left(1-p^e\right)\left(P-p^e\right) \tag{19}$$

Incumbent: $$\pi^I=D^I\left(P,p^e\right)\cdot P=P\left\{p^e-\frac{\left(p^e\right)^2}{2}+r-P\right\}. \tag{20}$$

ii) Entrant's Profits

The preceding analysis shows that the entrant makes zero profits under unbundled sales (equations (8) and (10)) and is profitable only under bundling. Figure 13 compares the entrant's profits under the two scenarios involving bundling, [Bundle, Entry in Good 1] (equation (15)) and [Bundle, Entry in Good 2] (equation (19)) for $1 \le r \le 10$. It demonstrates that the entrant makes higher profits under entry in good 2. Our numerical analysis shows that the same qualitative result applies for all $r \ge 1$. Therefore, the entrant will prefer to enter in good 2 rather than in good 1.

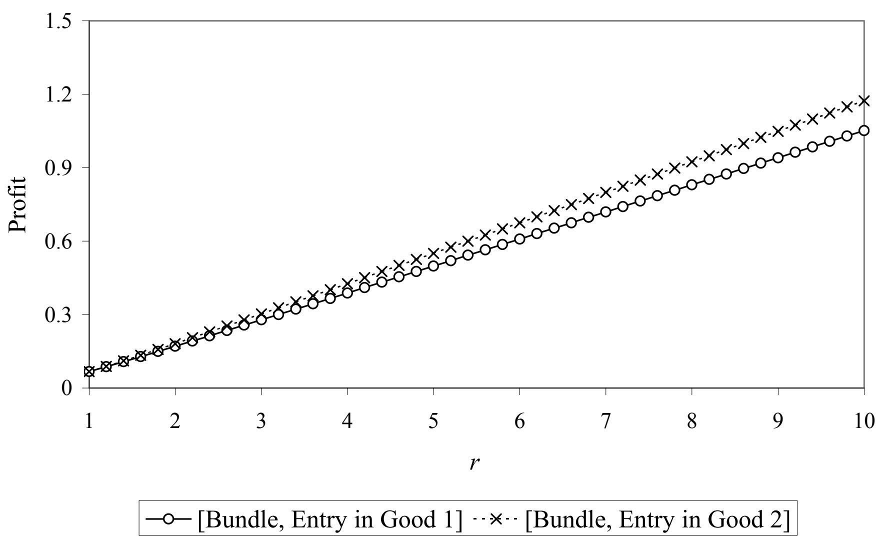

Figure 13. Entrant's profits under different entry options.

iii) Incumbent's Profits

Figure 14 represents the incumbent's maximum profits, given by equations (5), (6), (7), (9), (16), and (20), under various scenarios for $1 \leq r \leq 10$. For any r, the list of scenarios in order of profits is [Bundle, No Entry] > [No Bundle, No Entry] > [Bundle, Entry in Good 2] > [No Bundle, Entry in Good 2] > [Bundle, Entry in Good 1] > [No Bundle, Entry in Good 1]. From the numerical analysis, we verify that this relationship holds for all $r \geq 1$.

Therefore, we can infer that the incumbent's profits are highest with bundling and under no entry; if entry were to occur, the incumbent would prefer entry in good 2 to entry in good 1.

Proposition 3: *In a two-period set up, the incumbent maximizes the present value of its profits by choosing unbundled sales because that choice deters entry.*

Proof: In the first period, the incumbent is a monopolist and makes a decision about whether or not to bundle. Accordingly, its first period profits equal the profits under [Bundle, No Entry] or [No Bundle, No Entry] depicted in Figure 14. In the second period, the entrant decides whether or not to enter. If entry does not occur, the second-period profits for the incumbent will be the same as its first-period profits. Otherwise, the incumbent's profits will be those under [Bundle, Entry] or [No Bundle, Entry], depending on its choice in the first period. Table 1 summarizes possible two-period scenarios and each period's profits scenario that corresponds to Figure 14 (by Lemma 3, 'entry' in this analysis refers to 'entry in good 2').

It is evident from Figure 14 that the incumbent prefers scenario 1 to scenario 2 and scenario 3 to scenario 4, at any given discount rate. The figure also shows that the profits under scenario 1 are higher than those under scenario 3. Scenario 1, however, is not viable under the assumption of no fixed cost of entry, as discussed in section 3 of the paper: bundling induces entry whereas unbundling

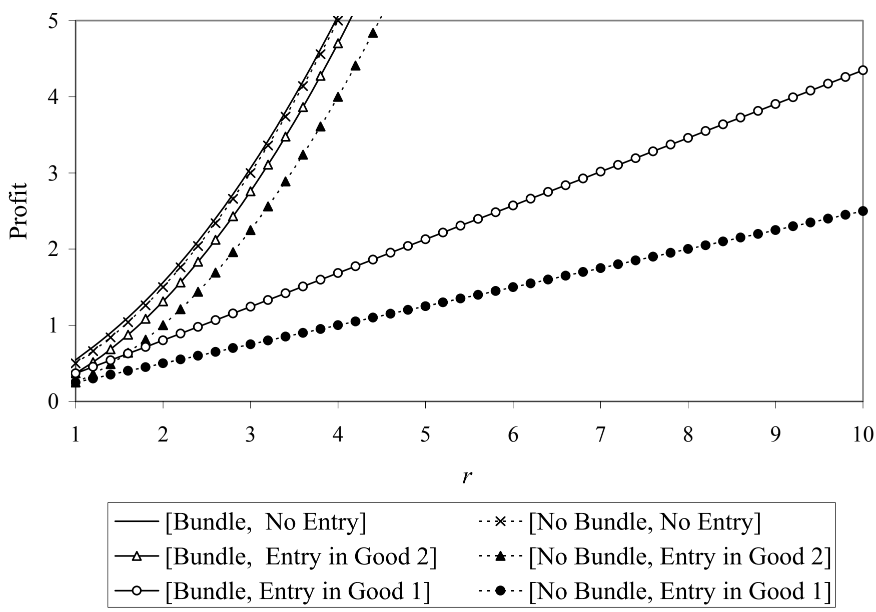

Figure 14. Incumbent's profits under various bundling and entry options.

Table 1. Incumbent's profits under two-period scenarios

Scenario	Incumbent's Choice in Period 1	Entrant's Choice in Period 2	Incumbent's Profits	
			Period 1	Period 2
1	Bundle	No Entry	[Bundle, No Entry]	[Bundle, No Entry]
2	Bundle	Entry	[Bundle, No Entry]	[Bundle, Entry]
3	No Bundle	No Entry	[No Bundle, No Entry]	[No Bundle, No Entry]
4	No Bundle	Entry	[No Bundle, No Entry]	[No Bundle, Entry]

deters entry under Bertrand competition. Therefore, the incumbent realizes the highest profits under scenario 3 by unbundling and deterring entry.

Proposition 4: *If $k < \pi^{e*}$, the optimal choice for the incumbent is to sell the two goods independently and deter entry; if $k \geq \pi^{e*}$, the optimal choice for the incumbent is to sell the goods as a bundle.*

Proof: Assume bundling. Then, without entry, the incumbent gains Δ_2 (the difference in profits between [Bundle, No Entry] and [No Bundle, No Entry]) in Figure 8 by choosing to bundle. However, bundling causes profit erosion because it induces entry when the cost of entry is low. This profit erosion is the difference in profits between [Bundle, No Entry] and [Bundle, Entry] in Figure 8. A comparison of the profit gain and loss shows that the profit erosion due to entry outweighs the gain from bundling. On the contrary, if the incumbent chooses unbundled sales, it gives up Δ_2 relative to bundling. However, the choice deters entry, and thus there is no further loss in profits. Ultimately, the net loss is smaller if the incumbent sells unbundled goods. We find consistent results for all $r \geq 1$ from our numerical analysis. Therefore, with low cost of entry, bundling cannot be the profit maximizing strategy for the incumbent. Rather, its optimal choice is unbundled sales and deterred entry; i) $k < \pi^{e*}$

ii) $k \geq \pi^{e*}$

There is no threat of entry due to the high entry cost: the fixed cost of entry exceeds what can be earned under Nash equilibrium by the entrant. As discussed in Proposition 2, the incumbent can maximize profits by bundling as a monopolist.