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The economics of crime and punishment: An analysis of optimal penalty

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Abstract

This paper demonstrates the optimality of a non-maximal penalty in a hierarchical enforcement structure. The penalty is chosen by the social planner to maximize the probability of monitoring and to minimize the probability of transgression. We compare the optimal penalty levels with and without ex post flexibility wherein the players are allowed to alter their decision after they observe the social planner's choice.

Keywords: Optimal penalty; Crime; Monitoring; Penalty

JEL classification: K42; C72; K21

1. Introduction

The economics of crime and punishment owes its underpinnings to the market-based approach proposed by Becker (1974). Some recent papers in this tradition are Eaton and White (1991), Schotter (1985), Benoit and Osborne (1995), Hylton (1996) and Philipson and Posner (1996). A distinct yet related set of studies has adopted a game theoretic approach. They include Graetz et al. (1986), Melumad and Mookherjee (1989), Chander and Wilde (1992), Grieson and Singh (1990), Russell (1990), and Bose (1995).

This paper adopts a hierarchical enforcement structure where the task of enforcement is delegated to a regulatory agency such as the EPA, IRS, or the traffic police that operates within the framework of government regulations or the judicial system. Like Chander and Wilde (1992) and Bose (1995), we demonstrate that the difference in the objectives of the social planner and the regulator contributes

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to the optimality of non-maximal fines. This result is in variance with the classic Beckerian conclusion that social welfare is strictly increasing in magnitude of the fine and, as a result, extreme penalties are socially optimal. However, while the optimality of non-maximal fines is a consequence of erroneous penalization in Bose (ibid), we demonstrate that non-maximal fines can be optimal even with error-free enforcement when the social planner's objective is suitably defined. In most papers, a social welfare function that includes both the harm suffered by victims and gains accrued by the perpetrators is maximized to determine the optimal penalty. In contrast, we model the social planner's objective as being the joint maximization of the probability of monitoring (and hence expected revenue) and minimization of the probability of transgression.

2. The model

Individuals, assumed to be identical, face the choice of whether to transgress. The gain from the transgression, R, although known to the individual, is unknown to the monitoring agency and is assumed to be distributed as a unit uniform distribution. Monitoring entails a cost, M, which although known to the agency, is unknown to the individual and is assumed to be distributed as a unit uniform distribution. The individual is aware that cost considerations prohibit the agency from monitoring constantly. However, if the individual transgresses when the agency is monitoring, s/he is caught with probability one and pays a penalty, P. The agency collects a predetermined portion of the penalty, θP, (where $0 < \theta \leq 1$) as a part of its operating revenue, while the remainder, $(1-\theta)P$, goes to the social planner. The pay-off matrix for the game is given by:

		Monitoring agency	
		Monitor	Not monitor
Individual	Transgress	$-P,\ \theta P - M$	$R,\ 0$
	Comply	$0,\ -M$	$0,\ 0$

From the perspective of the monitoring agency, the individual transgresses if the gain from the transgression exceeds an unknown threshold denoted by r. Based on the distributional assumptions regarding R, Pr{*individual transgresses*} $= \Pr\{R \geq r\} = 1 - r$ where Pr denotes probability. From the individuals' perspective, the agency monitors if monitoring cost falls below an unknown level, say m; therefore: Pr{*agency monitors*} $= \Pr\{M \leq m\} = m$. The individual's expected pay-off from the transgression is: $(-P)\cdot m + R\cdot(1-m)$. Since the gains from compliance are assumed to be zero, the individual transgresses if: $R \geq m \cdot P/(1-m) = r$, where the inequality follows from the distributional assumptions regarding R. This implies that monitoring probability is:

$$m = r/(P + r) \tag{1}$$

It can be similarly shown that the agency monitors if: $M \leq \theta \cdot P \cdot (1-r) = m$, which implies that compliance probability is:

$$r = (\theta P - m)/\theta P \tag{2}$$

Solving Eqs. (1) and (2):

$$\Pr\{individual\ complies\} = r^*(P,\theta) = P\left(\frac{2}{1 - \theta P - \theta P^2 + A} - 1\right)$$

$$\Pr\{agency\ monitors\} = m^*(P,\theta) = \frac{1 + \theta P + \theta P^2 - A}{2},$$

where $A = \sqrt{(1 + \theta P + \theta P^2)^2 - 4\theta P}$. The mixed strategy pair denoted by $(r^*(P, \theta), m^*(P, \theta))$ constitutes a Nash equilibrium. A few salient properties of the equilibrium are summarized below.

Lemma.

1*a*. $\underset{P \to 0}{\text{Limit}} \Pr\{r^*(\cdot)\} = 0$, $\underset{P \to 0}{\text{Limit}} \Pr\{m^*(\cdot)\} = 0$.

1*b*. $\underset{P \to +\infty}{\text{Limit}} \Pr\{r^*(\cdot)\} = 1$, $\underset{P \to +\infty}{\text{Limit}} \Pr\{m^*(\cdot)\} = 0$.

2*a*. $\dfrac{\partial r^*(\cdot)}{\partial P} > 0$, **2*b*.** $\dfrac{\partial m^*(\cdot)}{\partial P} \lessgtr 0$.

3. $\dfrac{\partial \underline{P}(\theta)}{\partial \theta} < 0$, *where* $\underline{P}(\theta) = \underset{P}{\operatorname{argmax}}\{m^*(P,\theta)\}$

4*a*. $\dfrac{\partial r^*(\cdot)}{\partial \theta} > 0$, **4*b*.** $\dfrac{\partial m^*(\cdot)}{\partial \theta} > 0$.

Fig. 1 contains the plots for the probabilities of compliance and monitoring as functions of the penalty level drawn under the assumption that $\theta = 0.5$.

3. Optimal penalty

What constitutes an optimal penalty depends on the social planner's objective. From Lemma 1b it is clear that P has to be infinitely large to ensure full compliance (i.e., $r^* = 1$). For certain types of crimes, where deterrence considerations are paramount, such extreme levels of penalty may be optimal. Consider now the other extreme where the social planner's sole objective is to maximize the expected revenue from monitoring:

Lemma 5. *For any given level of* θ*, the expected revenue from monitoring is maximized at* $P = \underline{P}$*, where* $\underline{P}$ *has been defined in Lemma 3. That is, the penalty level that maximizes monitoring probability also maximizes expected revenue from monitoring.*

In many social settings, the two polar objectives considered above – maximization of compliance without consideration for revenue (which would imply an infinitely large penalty) and maximization of expected revenue without consideration for compliance (which would imply a penalty of $\underline{P}$) – may be inappropriate. A more plausible objective is one that seeks to maximize the probability of monitoring and minimize the probability of transgression. The penalty that satisfies this objective is set out below:

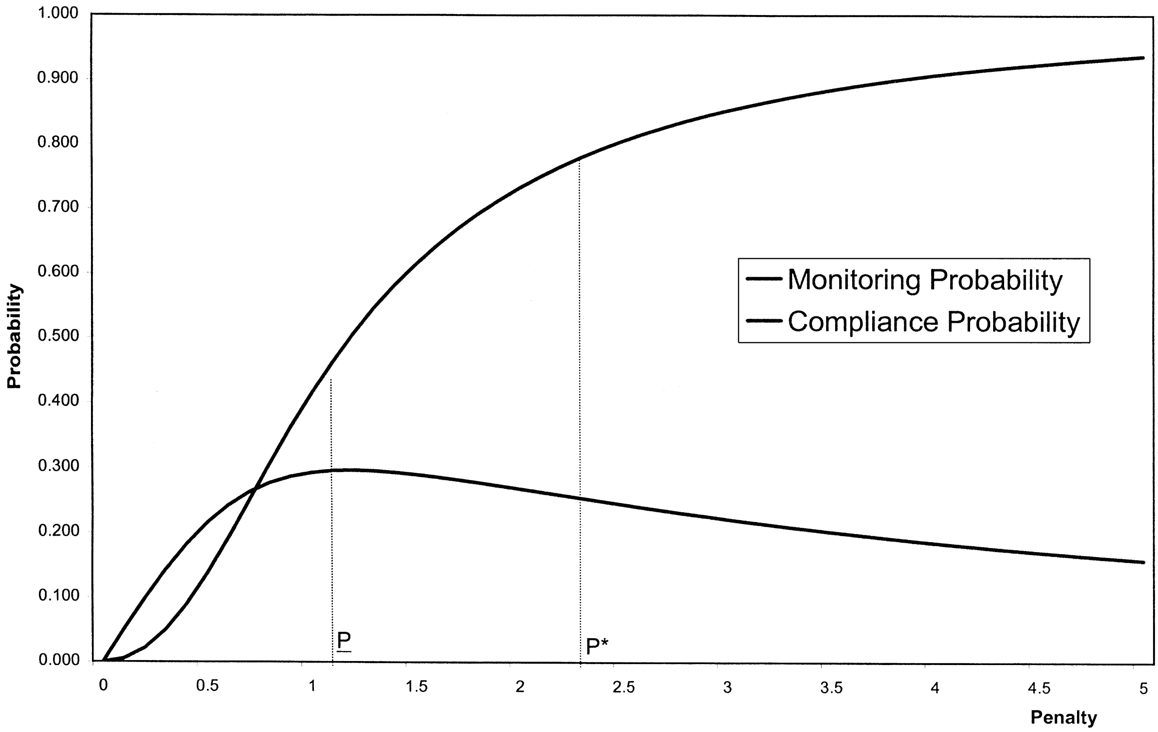

Fig. 1. Monitoring and compliance probability.

Proposition 1.

$$P^*(\theta) = \underset{P}{\operatorname{argmax}}\{m^*(P,\theta) \times r^*(P,\theta)\} = \frac{1}{6}\left(\sqrt{2}\sqrt{\frac{24 + \theta - \sqrt{\theta} \cdot \sqrt{12 + \theta}}{\theta}} + \sqrt{\frac{12 + \theta}{\theta}} - 1 \right).$$

Thus, P^* denotes the non-maximal level of penalty that maximizes the joint probability of compliance and monitoring. It can be readily verified that $P^*(\theta) > \underline{P}(\theta)$, for $0 \leq \theta \leq 1$.

4. Endogenous penalty

In this section we consider the setting where the penalty is endogenously determined in a three-player game, with the social planner being the additional player. That is, we allow for the interaction between the social planner's penalty choice and the agents' optimal strategy choices such that all three, the monitoring probability, the compliance probability, and the penalty level are jointly determined. As before the social planner's objective is to maximize the joint probability of compliance and monitoring, which using (1) and (2), can be written as:

$$\max_{P}\left\{\frac{r}{P + r} \times \frac{\theta P - m}{\theta P}\right\}.$$

Its first order condition implies:

$$2P \cdot m + m \cdot r - \theta P^2 = 0. \tag{3}$$

In the setting of endogenous penalty, the optimal choices and the optimal penalty are found by solving (1), (2), and (3) for the three unknowns, *r*, *m*, and *P*. The equilibrating conjectures and the optimal penalty are characterized below.

Proposition 2. *Under endogenous penalty:*

$$\hat{r}(\theta) = \frac{\sqrt{4+\theta} + \sqrt{\theta} - 2}{2\sqrt{\theta}},$$

$$\hat{m}(\theta) = \frac{2 + \sqrt{\theta} - \sqrt{4+\theta}}{2},$$

$$\hat{P}(\theta) = \frac{1}{\sqrt{\theta}}.$$

The relative magnitudes of the penalty levels under the various settings are summarized below.

Proposition 3.

3*a*: $\hat{P}(\theta) < P^*(\theta)$, *for all* $0 < \theta \leq 1$.
3*b*: $\hat{P}(\theta) > \underline{P}(\theta)$, *for all* $0 < \theta \leq 1$.

Proposition 3a implies that the penalty level needed to achieve the social planner's objective is lower in the endogenous setting which allows for the interaction between the agents' and social planner's choices. Numeric computation under various levels of θ show that in the setting of endogenous penalty the optimal level of penalty is 53 to 71 percent lower than that in the setting where the penalty is determined exogenously. The upshot of this result is: when the process of penalty choice and the penalty level are responsive to agents' behavior, the social planner's objective of minimizing transgressions and maximizing revenue can be achieved by a penalty level considerably smaller than the one required in a non-responsive environment.

Proposition 3b suggests that the socially optimal penalty, $\hat{P}(\theta)$, is higher than the level that maximizes the expected revenue from monitoring. This result implies that, if the choice of penalty were to be left in the hands of the monitoring agency, the chosen penalty level would be *lower* than that socially desired.

5. Conclusions

Penalties and fines are common enforcement tools in many transgressions such as tax evasion, traffic and parking violations, and antitrust offenses. Because fines often constitute a sizeable portion of monitoring agencies' operating budget, any meaningful analysis of optimal penalties cannot be based solely on deterrence considerations. It must also consider the likely impact of various penalty levels on the agencies' revenue. This paper addresses both these aspects of optimal penalty determination.

The optimal penalty is computed under two scenarios. In the first, the social planner determines the penalty that simultaneously maximizes the probability of monitoring *and* minimizes the probability of transgression. In the second scenario, the social planner has the same objective, but the penalty is endogenous. It affects each player's actions which, in turn, affect the penalty-level. It is shown that the social planner's objective is achieved with a lower penalty than in the first scenario. We also demonstrate that, if the choice of penalty were to be left in the hands of the monitoring agency, the chosen penalty level would be *lower* than that socially desired. Our paper's approach and results have important policy implications in a world characterized by tighter budgets and new approaches to crime deterrence.

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