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Complementary Goods: Prices and Consumer Welfare Under Duopoly and Monopoly

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ABSTRACT *We examine prices, profits, and consumer surplus for differentiated complementary goods under duopoly and a multi-product monopoly. We find that little can be said about the relative magnitudes of prices of the components of a system of complementary goods under the alternative market structures. Although demand complementarity can lead to lower prices for either the primary or the secondary good under monopoly, both prices are not necessarily lower. The results unique to this paper are that, when two complementary goods form a system, the system price is unambiguously lower and consumer surplus and profits are higher under a multi-product monopoly.*

Key Words: Complements; Multi-Product Monopoly.

JEL Classifications: D42, L11, L40.

1. Introduction

Consider the following scenario: two firms, each manufacturing a differentiated good, merge to form a single multi-product monopoly. Will the post-merger prices be higher? If there are no merger-induced efficiencies and if the two goods are substitutes, the answer to this question is almost always 'yes.' However, the picture is far less clear when the goods are complements. As early as 1838, Cournot had shown that a multi-product monopolist charges lower prices for complements than goods with independent demands (see also Tirole, 1988: 70). However, does that imply *all* complementary good prices of a multi-product monopolist are lower than the equilibrium prices under duopoly? What is the impact of demand complementarity on consumer welfare? Surprisingly little

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research has systematically examined these issues. Yet goods with complementary demands are ubiquitous in many sectors of the economy, particularly, in the high-tech industries (see Shy, 2001): Zip drive and Zip disks, compatible server hardware and software, Palm PDA and Palm-compatible applications, Adobe Acrobat Reader and Acrobat Writer are just a few such examples in the computer technology-related industries.

In this paper, we examine prices, profits, and consumer surplus for differentiated complementary goods under duopoly and a multi-product monopoly. We find that very little can be said about the relative magnitudes of the products' individual prices under the alternative market structures of duopoly and monopoly. In particular, demand complementarity can lead to lower prices for either of the two goods under a monopoly, but both prices are not necessarily lower.

This paper focuses on firms' pricing decisions when complementary goods form a system. In the case of these goods, the consumer purchases a primary good (e.g., Zip drive) that typically uses a fixed or variable number of units of a complementary good (e.g., Zip disks). Thus, the primary and secondary goods form a system of complementary goods. The system price – which is the effective price faced by a consumer – is the sum of the price of the primary good and the total cost of the units of the complementary good consumed per unit of the primary good. The system price is typically different from the simple sum of the component prices, since the number of the secondary goods consumed per unit of the primary good may be different from unity. The results unique to this paper are that, when the two complementary goods form a system, the *system price* is unambiguously lower and consumer surplus and profits are higher under a multi-product monopoly compared to duopoly.

2. Prior Research on Pricing of Complements

There are a few studies that have directly examined the issue of demand complementarity and its impact on firms' pricing decisions. In this section, we briefly review these studies to delineate our contribution to the existing literature.

The paper most directly related to ours is the one by Davis and Murphy (2000). This paper examines demand complementarity between Windows operating system and Internet Explorer (IE) and argues that the zero pricing of IE can be explained by the fact that Web usage stimulates the demand for Windows. In their paper, Davis and Murphy set out a simple linear demand model with two goods to demonstrate that, because of complementary demand, a profit-maximizing firm may price one of the two goods below marginal cost. Much of the economic intuition for the main results in the Davis and Murphy paper carry over to our study.

A few studies have explicitly considered the pricing decisions of a multi-product monopolist selling complementary goods. For example, Tirole (1988: 70) examines the optimality conditions for a multi-product monopolist. His analysis suggests that the multi-product monopolist's prices for *each* good will be lower than the prices charged by separate firms each producing a single good. Assuming two goods, Tirole's expression for the monopolist's first order condition for the i^{th} good can be written as (denoting the i^{th} good's price and marginal cost as p_i and $C_i^{'}$):

$$\frac{p_i - C_i^{'}}{p_i} = \frac{1}{\varepsilon_{ii}} - (p_i - C_i^{'})\theta_{ij}$$

where $\frac{p_i - C_i^{'}}{p_i}$ is the Lerner index, $\varepsilon_{ii} = -\left(\frac{\partial D_i}{\partial p_i} \cdot \frac{p_i}{D_i}\right)$ is the own price elasticity, $\varepsilon_{ij} = -\left(\frac{\partial D_i}{\partial p_j} \cdot \frac{p_j}{D_i}\right)$ is the cross price elasticity, and $\theta_{ij} = \frac{D_j \varepsilon_{ij}}{p_i D_i \varepsilon_{ii}}$. If the two goods are complements then $\theta_{ij} > 0$. Also, if separate firms produced the two goods, then each firm's first order condition would be simply: $\frac{p_i - C_i^{'}}{p_i} = \frac{1}{\varepsilon_{ii}}$, which means that the Lerner index would be equal to the inverse of the own elasticity of demand. In discussing the optimality condition of the multi-product monopolist, Tirole (1988) writes: '…for complements…the inverse of the own elasticity of demand exceeds the Lerner index for *each* good' (70, emphasis added). Because a higher Lerner index implies a higher price, Tirole's analysis suggests that *each* complementary good's price under a multi-product monopoly would be lower than prices charged by individual firms. It is easy to see, however, that need not be the case. As an apparent matter, if the optimal price of one of the two complementary goods is below the good's marginal cost, then $(P_i - C_i^{'}) \cdot \theta_{ij} < 0$ and the Lerner index for that good will exceed the inverse of own elasticity. This implies that, contrary to Tirole's claim, the price of *each* complementary good under multi-product monopoly need not be lower than the prices charged by each individual firm.

Importantly, one can demonstrate through a simple numeric example that a multi-product monopolist may actually charge a higher price for one of the complementary goods than firms producing the goods separately even if the monopolist's optimal price for each good is higher than the marginal cost. The latter result, demonstrated below, is driven by differences in elasticities evaluated at the equilibrium duopoly and multi-product monopoly prices. More specifically, the Tirole conclusion is erroneous because, although the elasticity *expression* (i.e., $\frac{1}{\varepsilon_{ii}}$) is identical in both the multi-product monopolist's and each of the individual firm's first order condition, their *values* are different since they are evaluated at different prices.

We assume linear demand and that the demands for the two goods are: $q_1 = A - \alpha_1 p_1 - \alpha_2 p_2$ and $q_2 = B - \beta_1 p_1 - \beta_2 p_2$. We assume the following parameter values: $A = 3$, $\alpha_1 = 6$, $\alpha_2 = 0.1$, $B = 7$, $\beta_1 = 3$, and $\beta_2 = 5$. The marginal cost values are set at $c_1 = 0.1$ and $c_2 = 0.2$. In this example, the prices charged by the multi-product monopolist are found to be 0.156 and 0.753, while the prices charged by the two duopolists, under Bertrand competition, are 0.294 and 0.712, respectively. Thus, the monopolist charges a higher price for the second good compared to duopoly even when all equilibrium prices exceed the corresponding margin costs and the two goods are complements.[1] Indeed, nothing can be said about the relative magnitudes of the individual prices under multi-product monopoly vis-à-vis duopoly under the general framework of complementary goods.

A few studies have examined firms' pricing decisions when complementary goods form a system. Shy (1996) compares the prices for goods sold by two separate firms with the system price charged by a multi-product monopolist. His analysis assumes, however, that the two goods are perfect complements (that is, both goods are always consumed in fixed one-to-one proportion) – which implies that the system price is simply the sum of the individual prices. He then demonstrates that the sum of multi-product monopolist's optimal prices will be lower that the sum of the prices under duopoly.

We extend Shy's analysis to the case of imperfect complementarity.[2] Additionally, unlike Shy, we do not assume linearity of demand. We introduce the concept of system prices in this more general framework of imperfect complementarity under nonlinear demand and demonstrate that that the monopolist's *system price* is unambiguously lower than the system price that would have prevailed if the components of the system were produced by two independent firms. Interestingly, however, we find that the difference in the system prices under monopoly and duopoly is independent of the strength of the complementarity of the two goods. That, however, is not true for consumer surplus – the percentage difference in consumer surplus under monopoly and duopoly is an increasing function of the strength of the complementarity of the two goods. In the last section of the paper we provide a numerical example to quantify the difference in prices, profits and consumer surplus under the alternative market structures. These results suggest that entry into a market dominated by a multi-product monopolist may not necessarily lead to lower prices and enhance consumer welfare when the products at issue form a system of complementary goods.

3. The Model

We assume that there are two differentiated goods with complementary demands: good 1, whose price is denoted by p_1, is the primary good, and good 2, with a price of p_2, is the secondary good. For each unit of good 1, consumers use g units of good 2; thus, the system price is: $P = p_1 + g \cdot p_2$. The quantity of the primary good demanded is denoted by Q_1; consequently, the demand for the secondary good is: $Q_2 = g \cdot Q_1$. For some consumer products, g, the number of units of good 2 consumed per unit of good 1, is fixed (e.g., as a result of product design); however, in most settings, g will be a declining function of price. Therefore, we assume that $g = g(p_2) > 0$, with $g'(p_2 \leq 0)$. We further posit that the consumers' demand for the primary good is influenced by the cost of buying the system (e.g., consumers care not only about the price of the Zip drive but also of Zip disks in deciding whether or not to buy a Zip drive) and not merely by price of the primary good, p_1. In other words, the demand for the primary good is a function of the system price: $Q_1 = f(P)$. The structure of the demand for the primary and the secondary good outlined above can be derived from a framework of consumer's utility maximization. This framework is set out in Appendix 1.

We make the following additional assumptions throughout the paper:

A1: Primary good demand is a decreasing function of system price, that is, $f'(P) < 0$.

A2: The system price is an increasing function in each of the two component prices; in particular,

$$\frac{\partial P}{\partial p_1} > 0 \text{ and } \frac{\partial P}{\partial p_2} = (g + p_2 \cdot g') > 0.$$

A3: The marginal cost of producing the primary good is fixed and is equal to $c \geq 0$ while the marginal cost of the secondary good is zero.

Assumption A1 seems eminently reasonable since it simply requires the demand for the primary good to be a decreasing function of the system price (it also follows from the utility maximization problem set out in Appendix 1). The first part of Assumption A2, that is, $\frac{\partial P}{\partial p_1} > 0$, obviously holds by the definition of system price since $\frac{\partial P}{\partial p_1} = 1$. However, because we have assumed $g'(p_2) \leq 0$, the expression $(g + p_2 \cdot g')$ can be negative, a possibility that is precluded by A2. Thus, the condition $\frac{\partial P}{\partial p_2} = (g + p_2 \cdot g') > 0$ in A2 rests on the premise that a higher component price must lead to a higher system price. Additionally, assumption A2 is necessary for demand complementarity; in particular, observe that $\frac{\partial Q_1}{\partial p_2} = \frac{\partial Q_1}{\partial P} \cdot \frac{\partial P}{\partial p_2} = f' \cdot (g + p_2 \cdot g')$. By A1 since $f' < 0$, $\frac{\partial Q_1}{\partial p_2} < 0$, only if $(g + p_2 \cdot g') > 0$. Assumption A3 is likely to be true for software products and information goods, which typically require substantial upfront fixed costs (e.g., R&D costs) but each additional unit is produced at little or virtually no cost.

In what follows, we first set out the optimization problems under duopoly; the first firm is assumed to produce only good 1 and the second only good 2. We derive the two firms' first order conditions and examine the Nash equilibrium prices. We then turn to the choice problem of the monopolist producing both the goods. The key results of the paper follow from the comparison of the prices, profits, and consumer surplus under the two market structures.

3.1. The Choice Problems under Duopoly

Following Shapiro's (1989: 346) exposition of the firm's choice problem for differentiated goods, we set out the duopolists' optimization problems as:

$$\underset{p_1}{Max}\, \pi_1 = Q_1(P) \cdot (p_1 - c)$$

and

$$\underset{p2}{Max}\, \pi_2 = Q_1(P) \cdot g(p_2) \cdot p_2.$$

Differentiating each firm's profit functions by the respective price yields the two first order conditions:

$$Q_1 + (p_1 - c) \cdot f'(P) = 0 \tag{1}$$

$$(g + g' \cdot p_2) \cdot (Q_1 + p_2 \cdot f'(P) \cdot g) = 0. \tag{2}$$

However, since by Assumption A2, $(g + p_2 \cdot g') > 0$, firm 2's first order condition reduces to:

$$Q_1 + p_2 \cdot f'(P) \cdot g = 0. \tag{2a}$$

The Nash equilibrium prices occur at the intersection of the firms' reaction functions; therefore, it follows from the comparison of the (1) and (2a) that, at the equilibrium prices denoted by p_1^0 and p_2^0, the following must be true:

$$\left(p_1^0 - c\right) = g^0 \cdot p_2^0, \tag{3}$$

where $g^0 = g(p_2^0)$. Equation (3) has three interesting implications. First, the optimal level of the system price under duopoly is independent of $g(\cdot)$, which represents the strength of complementarity between the two goods. In particular,

$$P^0 = \left(p_1^0 + g^0 \cdot p_2^0\right) = \left(2p_1^0 - c\right). \tag{4}$$

Second, despite the fact that the two firms' marginal costs are different, the two firms' profits at the Nash equilibrium prices are identical; that is:

$$\pi_1\left(p_1^0, p_2^0\right) = \pi_2\left(p_1^0, p_2^0\right) = \left(p_1^0 - c\right) \cdot Q_1\left(P^0\right). \tag{5}$$

And finally, as a result of (5), the sum of the two firms profits evaluated at the Nash equilibrium prices is a function of the system price only and is also independent of $g(\cdot)$. In particular:

$$\pi^0\left(p_1^0, p_2^0\right) = \pi_1\left(p_1^0, p_2^0\right) + \pi_2\left(p_1^0, p_2^0\right) = \left(P^0 - c\right) \cdot Q_1\left(P^0\right). \tag{6}$$

3.2. The Multi-product Monopolist's Choice Problem

The monopolist producing both goods 1 and 2 solves the following problem:

$$\underset{p_1, p_2}{Max}\, \pi = Q_1(P) \cdot (p_1 + p_2 \cdot g - c). \tag{7}$$

The monopolist's two first order conditions are:

$$Q_1 + (P - c) \cdot f'(P) = 0. \tag{8a}$$

$$\left(g + g' \cdot p_2\right) \cdot \left(Q_1 + f'(P) \cdot (P - c)\right) = 0. \tag{8b}$$

By Assumption A2, $(g + p_2 \cdot g') > 0$; as a result, (8b) reduces to: $Q_1 + (P - c) \cdot f'(P) = 0$, which is identical to the first order condition, (8a). Thus, the monopolist's problem in (7) does not have a unique solution; any set of optimal prices, denoted by p_1^* and p_2^*, that satisfy condition 8a maximizes the monopolist's profits:[3]

$$f(p_1^* + g^* \cdot p_2^*) + (p_1^* + g^* \cdot p_2^* - c) \cdot f'(p_1^* + g^* \cdot p_2^*) = 0, \tag{9}$$

where $g^* = g(p^*)$. Equation (9) represents the locus of monopolist's optimal prices. An important implication of (9) is that nothing can be said about the relative magnitudes of the individual optimal prices under duopoly and monopoly; that is: $p_i^* \lesseqgtr p_i^0, i = 1, 2$. Secondly, as has been noted by previous authors, one of the two optimal prices of a multi-product monopolist can be below the marginal cost.

Let us now use (7) to reformulate the monopolist's choice problem in an equivalent form of choosing a system price:

$$\underset{P}{Max}\,\pi = Q_1(P)\cdot(P-c) \tag{10}$$

which yields the first order condition

$$Q_1 + (P-c)\cdot f'(P) = 0 \tag{11}$$

Assuming $\frac{\partial^2 \pi}{\partial P^2} = 2f'(P) + (P-c)\cdot f''(P) < 0$, (11) is solved by a unique and strictly interior P^*. It is important to recognize that any set of optimal prices, p_1^* and p_2^*, that satisfy (9) will also yield the same P^* that solves (11); that is, $P^* = p_1^* + g^* \cdot p_2^*$ (since equations (9) and (11) are identical).

It is also readily verifiable that, as in the case of the system price under duopoly, the multi-product monopolist's optimal system price, P^*, and optimal profit, $\pi(P^*)$, are both independent of strength of complementarity of the two goods, represented by g.

We now state the key results of the paper. The proofs of all the propositions are presented in Appendix 2.

Proposition 1: The optimal system price under monopoly is lower than that under duopoly; that is,

$$P^* \equiv \left(p_1^* + g\cdot p_2^*\right) < \left(p_1^0 + g\cdot p_2^0\right) \equiv P^0.$$

Proposition 2: The multi-product monopolist's optimal profit exceeds the sum of the duopolists' profits evaluated at Nash equilibrium prices; that is:

$$\pi\left(P^*\right) > \pi_1\left(P^0\right) + \pi_2(P^0).$$

The economic intuition for the foregoing results is that a multi-product monopolist can internalize the benefits of a price reduction of complementary goods: for example, a lower price for good 1 induces higher sales of not only good 1 but also of good 2, increasing the profits of the integrated firm that sells both goods. By contrast, when separate firms sell the two goods, the benefits of complementary demands are not fully captured by the individual firms.

We now turn to the examination of consumer surplus (CS). For the good 1, the analysis is relatively straight forward, since its demand is a function of system price only. Noting that the functional form for the first good, $Q_1(P)$, is identical under duopoly and monopoly, the difference in CS under the two market structures is:

$$\Delta CS_1 = \int_{P^*}^{P^0} Q_1(P)dP\cdot \tag{12}$$

The demand for the second good is a function of both prices. Under duopoly, with Nash conjectures, the demand for the second good is: $Q_2 = g(p_2)Q_1(p_1^0, p_2)$. Since, by assumption $g(p_2) > 0$, the total number of the secondary goods demanded is equal to zero only when no primary units are consumed. Let $\hat{p}_2$ denote that value of the second good's price at which demand for the first good is zero, that is, $Q_1(p_1^0, \hat{p}_2) = 0$. Then, the CS for good 2 is:

$$CS_2^0 = \int_{p_2^0}^{\hat{p}_2} g(p_2) Q_1(p_1^0, p_2) dp_2 \cdot \tag{13}$$

To determine the consumer surplus under monopoly, recall that there are an infinite number of optimal prices, p_1^* and p_2^*, that maximize the monopolist profits. Thus, to meaningfully compare the CS for good 2 under the two market structures, we assume that monopolist's optimal price for good 1 is equal to the duopoly equilibrium level, that is, $p_1^* = p_1^0$. This assumption implies that, the value of $\hat{p}_2$ is identical under monopoly and duopoly. Thus, the difference in CS for good 2 under the two market structures is:

$$\Delta CS_2 = \int_{p_2^*}^{p_2^0} g(p_2) Q_1(p_1^0, p_2) dp_2 \cdot \tag{14}$$

Proposition 3: The consumer surplus for each good under monopoly is higher than the consumer surplus under duopoly with Nash equilibrium prices; that is: $\Delta CS_1 > 0$, $\Delta CS_2 > 0$.

The economic intuition for the above result follows from the fact that the optimal system price under monopoly is unambiguously lower than that under duopoly. Proposition 3 suggests that, when complementary goods form a system, consumer welfare may be impaired if the market structure is transformed from a multi-product monopoly to one where two separate firms produce the two goods.

4. A Numeric Example

In order to illustrate and quantify the results stated in Propositions 1–3, we now turn to a numeric example using simple functional form assumptions. We assume that the marginal costs of both goods are zero, and the demand functions for good 1 and 2 are:

$$\begin{aligned} Q_1 &= f(P) = 1 - P \\ Q_2 &= Q_1 \cdot g(p_2) = Q_1 \cdot (a - p_2) \end{aligned} \tag{15}$$

where $P = p_1 + p_2 \cdot (a - p_2)$ is the system price. Note that each good's demand is a decreasing function of its own and cross price (implying complementarity) as long as $a - 2p_2 > 0$. With this demand specification, the parameter a reflects the strength of complementarity between the two goods.

Under duopoly, the loci of the two firms first order conditions are shown in Figure 1. The Nash equilibrium occurs at the intersections of these loci. It is clear from Figure 1 that there are multiple equilibriums in this Nash game. However, we can rule out the solutions that do not satisfy the complementarity condition, $a - 2p_2 > 0$. Thus, we are left with a single set of prices that maximize both independent firms' profits:[4]

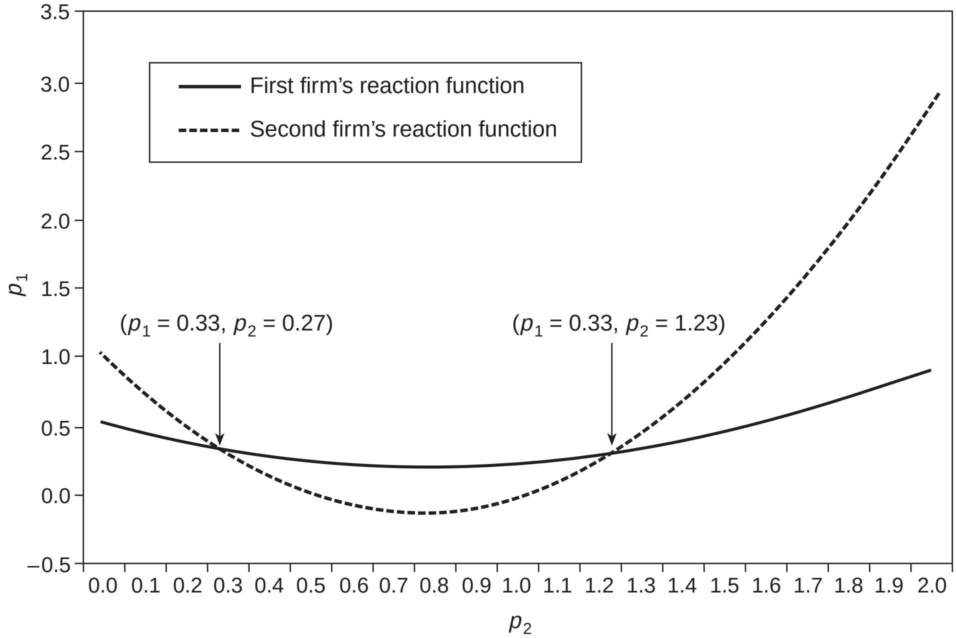

Figure 1. The Loci of the Two Firms First Order Conditions Under Demand Complementarity with a = 1.5.

$$p_1^0 = \frac{1}{3}, p_2^0 = \frac{1}{6}\cdot\left(3a - \sqrt{9a^2 - 12}\right), \text{ and } P^0 = \frac{2}{3}. \tag{16}$$

The corresponding optimal quantities and profits of the two firms are given by

$$Q_1^0 = \frac{1}{3}, Q_2^0 = \frac{1}{18}\cdot\left(3a + \sqrt{9a^2 - 12}\right), \text{ and } Q^0 = \frac{1}{18}\cdot\left(6 + 3a + \sqrt{9a^2 - 12}\right) \tag{17}$$

$$\pi_1^0 = \pi_2^0 = \frac{1}{9} \text{ and } \pi^0 = \frac{2}{9}\cdot \tag{18}$$

Under monopoly, the optimal system price is determined by solving the first-order condition (11), which yields $P^* = \frac{1}{2}$. As discussed in the previous section, a unique pair of optimal prices, p_1^* and p_2^*, does not exist; instead, any prices that satisfy $P^* = p_1^* + p_2^* \cdot g^* = \frac{1}{2}$ are profit maximizing under monopoly. Based on this system price the profit under monopoly is $\pi^* = \frac{1}{4}$.

We can now compare the system prices and profits under duopoly and monopoly. As noted in the previous section, the differences in system prices and profits are independent of parameter *a*. In particular,

$$\Delta P = P^0 - P^* = \frac{1}{6} \tag{19}$$

$$\Delta\pi = \pi^* - \pi^0 = \frac{1}{36}. \tag{20}$$

In percentage terms, the prices are 33% lower, and the profits are 11% higher under monopoly.

The difference in consumer surplus between duopoly and monopoly is given by

$$\Delta CS = \Delta CS_1 + \Delta CS_2. \tag{21}$$

The demand for the first good is a function of the system price; therefore, the consumer surplus difference for the first good can be calculated as:

$$\Delta CS_1 = \int_{P^*}^{P^0} Q_1 dP = \int_{1/2}^{2/3} (1-P)dP = \frac{1}{8} - \frac{1}{18} = \frac{5}{72}. \tag{22}$$

The demand for the second good is a function of the system price as well as the price of the second good. Using Nash conjectures, we calculate the consumer surplus for the second good under duopoly by holding the price of the first good at its equilibrium level, $p_1^0 = \frac{1}{3}$. First, we find the level of p_2, at which the demand for the first good and, consequently, for the second good is zero:[5]

$$\hat{p}_2 \cdot g(\hat{p}_2) = \hat{P} - p_1^0 = 1 - \frac{1}{3} = \frac{2}{3} \Rightarrow \hat{p}_2 = \frac{a}{2} - \frac{\sqrt{9a^2 - 24}}{6}. \tag{23}$$

Using $\hat{p}_2$, we then calculate the consumer surplus for the second good under duopoly as:

$$CS_2^0 = \int_{p_2^0}^{\hat{p}_2} Q_2 dp_2 = \frac{1}{36} + \frac{10a - 3a^3}{216} x + \frac{3a^3 - 8a}{216} y, \tag{24}$$

where $x = \sqrt{9a^2 - 12}$ and $y = \sqrt{9a^2 - 24}$.

To calculate the consumer surplus for the second good under monopoly, we assume that the monopolist charges a price equal to the Nash equilibrium price for the first good, i.e. $p_1^* = p_1^0 = 1/3$ (which implies that $\hat{p}_2$ is identical under monopoly and duopoly). The corresponding optimal price of the second good, which is solved using the relation $p_1^* = p_2^* \cdot g^* = \frac{1}{2}$, is given by $p_2^* = \frac{1}{6} \cdot \left(3a + \sqrt{9a^2 - 6}\right)$. We can now calculate the consumer surplus for the second good under monopoly as:

$$CS_2^* = \int_{p_2^*}^{\hat{p}_2} Q_2 \, dp_2 = \frac{1}{16} + \frac{11a - 3a^3}{216} z + \frac{3a^3 - 8a}{216} y, \tag{25}$$

where $z = \sqrt{9a^2 - 6}$.

The differences in the consumer surplus of the second good and the overall system are:

$$\Delta CS_2 = CS_2^* - CS_2^0 = \frac{5}{144} + \frac{11a - 3a^3}{216} z - \frac{10a - 3a^3}{216} x \quad (26)$$

$$\Delta CS = \Delta CS_1 + \Delta CS_2 = \frac{5}{48} + \frac{11a - 3a^3}{216} z - \frac{10a - 3a^3}{216} x. \quad (27)$$

For any positive value of a, ΔCS is positive, which implies that the consumer surplus is higher under monopoly compared to that under duopoly. Furthermore, the difference in consumer surplus, ΔCS, is a monotonically decreasing function of the parameter a, which reflects the strength of complementary between the two goods, and in the limit as a increases, we have $\underset{a\to\infty}{Limit}\ \Delta CS = \frac{5}{36}$.

The percentage difference in CS between monopoly and duopoly, however, is an increasing function of the parameter a. In fact, it can be verified that:

$$\underset{a\to\infty}{Limit} \frac{\Delta CS}{CS^*} = \frac{5}{9}, \quad (28)$$

where CS^* denotes the consumer surplus under monopoly. It can be readily verified that $\frac{\Delta CS}{CS^*}$ increases with a but levels off at 55.6%.

5. Concluding Comments

In this paper, we have examined prices, profits, and consumer surplus for differentiated complementary goods under duopoly and a multi-product monopoly. We find that very little can be said about the relative magnitudes of the component prices under the alternative market structures of duopoly and monopoly under the general setting of complementary goods. Although demand complementarity can lead to lower prices for either the primary or the secondary good under monopoly, both prices are not necessarily lower. The results unique to this paper are that the *system price* is unambiguously lower and consumer surplus and profits are higher under a multi-product monopoly. Numeric analysis suggests that, depending on the functional form of demand, the difference in system prices and consumer surplus under the two market structures can be substantial. These results suggest that entry into a market dominated by a multi-product monopolist may not necessarily lead to lower prices and enhance consumer welfare when the products at issue are complements.

Notes

1. The error in Tirole's analysis becomes transparent by noting that the own price elasticities for the two goods, evaluated at the optimal prices, are quite different under the two market structures. For example, the own-price elasticity for good 1 (ε_{11}) evaluated at the optimal prices for the multi-product monopolist is 0.469, while at duopoly prices it is 1.515. The corresponding figures for good 2 are 1.359 and 1.391.
2. A reviewer correctly points out that, in the framework of our paper, neither part of the system has value without the other; and to that extent they are perfect complements. However, the structure is different from that of Shy because the components of the system need not be used in fixed one-to-one proportion.

3. The fact that the monopolist's choice problem does not have a unique solution also follows from noting that $\pi_{11} \cdot \pi_{22} - (\pi_{12})^2 = 0$, where $\pi_{ij} = \dfrac{\partial^2 \pi}{\partial p_i \partial p_j}$.
4. In addition, the second order sufficient condition of the second firm requires $a > 2/\sqrt{3}$.
5. Two other solutions of the equation $\hat{p}_2 \cdot g(\hat{p}_2) = \dfrac{2}{3}$ do not satisfy the complementarity condition.
6. The usage of a quasilinear utility is common in the economics literature; see, for example, Varian, p. 154, 164, 419.

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Appendix 1. The Derivation of the Demand Structure for the Two Goods

Typically the purchase decisions of the primary and the secondary goods are not made simultaneously by a consumer. It is common, for example, for the consumer to decide whether or not to buy the primary good based on the likely future consumption level of the secondary good; after the primary good purchase has been made, the consumer will typically decide on the actual level of the consumption of the secondary good based on its prevailing price. For example, after the consumer has purchased a printer, the intensity of usage of the printer will depend on the price of the toner-cartridge.

With the foregoing in mind, we set out the consumer's choices in a simple two-period problem, where the second period is the terminal one. In the first period, the consumer chooses the number of units of the primary good, while in the second period the consumption intensity (denoted by g) of the secondary good is chosen based on its price. By backward induction we begin with the second period. Let S denote the level of savings for period 2 and the savings per unit of the primary good is denoted by $s = g \cdot p_2$; thus, $g = \frac{s}{p_2}$, that is, the level of usage of the secondary good is a declining function of its price, p_2, that is, $g'(p_2) < 0$.

We posit that the consumer derives utility from the consumption of the primary good and not from the secondary good directly – for example, the purchaser of a printer gets no direct utility from the consumption of the printer cartridge. In period 1, the consumer's choice problem is:[6]

$$\underset{Q_1}{Max}\, U(Q_1) + h \text{ subject to } Y - S = p_1 \cdot Q_1 + h$$

where Y denotes the consumer's income and h denotes the numeriare good. Since, $S = Q_1 \cdot p_2 \cdot g$, the consumer's problem can be re-written as (note S is not a choice variable since its level is determined by Q_1 and the expected levels of p_2 and g):

$$\underset{Q_1}{Max}\, U(Q_1) + Y - p_2 \cdot Q_1 \cdot g - p_1 \cdot Q_1$$

The first order condition of utility maximization is given by:

$$U'(Q_1) - (p_1 + g \cdot p_2) = 0$$

or

$$U'(Q_1) - P = 0$$

which yields the demand for the primary good as: $Q_1(P)$. It clear also from the differentiation of the first order condition that $Q_1{}'(P) < 0$.

Appendix 2. Proofs of the Propositions

Proof of Proposition 1: Using (11) in main body of the paper, the monopolist's first order condition for the choice of P, evaluated at the Nash equilibrium system price, P^0, is:

$$\left.\frac{\partial \pi}{\partial P}\right|_{P^0} = \left(P^0 - c\right)\cdot f'\left(P^0\right) + Q_1\left(P^0\right)$$
$$= \left(p_1^0 - c\right)\cdot f'\left(P^0\right) + Q_1\left(P^0\right) + g\left(p_2^0\right)\cdot f'\left(P^0\right)\cdot p_2^0.$$

But by (1) in main body of the paper, $\left(p_1^0 - c\right)\cdot f'(P^0) + Q_1(P^0) = 0$; thus:

$$\left.\frac{\partial \pi}{\partial P}\right|_{P^0} = g\left(p_2^0\right)\cdot f'\left(P^0\right)\cdot p_2^0 < 0.$$

Since, $\pi(P)$ is assumed to be strictly concave in P, it follows from the two results $\left.\frac{\partial \pi}{\partial P}\right|_{P^0} < 0$ and $\left.\frac{\partial \pi}{\partial P}\right|_{P^*} = 0$ that that $P^* < P^0$. □

Proof of Proposition 2: First note that the sum of the two duopolists' profits, $\pi_1(P) + \pi_2(P)$, is identical in functional form to the monopolist's profit, $\pi(P)$, i.e., $\pi(P) = \pi_1(P) + \pi_2(P)$. We know that $\pi(P)$ is maximized at P^* and that $P^0 \neq P^*$; therefore it must be true that $\pi(P^*) > \pi(P^0)$. □

Proof of Proposition 3: Since the first good's demand function, $Q_1(P)$, is identical in functional form under duopoly and monopoly, the difference in consumer surplus under the two market structures for good 1 is:

$$\Delta CS_1 = \int_{P^*}^{P^0} Q_1(P)\,dP.$$

But we know that $Q_1(P)$ is strictly positive for all values of $P \subset [P^*, P^0]$ and by Proposition 1 $P^* < P^0$; thus it follows that $\Delta CS_1 > 0$. For the second good,

$$\Delta CS_2 = \int_{P_2^*}^{P_2^0} g(p_2) Q_1\left(p_1^0, p_2\right) dp_2.$$

Since: (a) by assumption, $p_1^* = p_1^0$ and (b) by Assumption A2 $\frac{\partial P}{\partial p_2} > 0$, the result $P^* < P^0$ implies that $p_2^* < p_2^0$. Also, the demand for the second good, $g(p_2) Q_1(p_1^0, p_2)$, is positive for all values of $p_2 < \hat{p}_2$. Hence $\Delta CS_2 > 0$. □