RISK PREFERENCE ESTIMATION IN THE NONLINEAR MEAN STANDARD DEVIATION APPROACH

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Risk preferences and technology are jointly estimated in the nonlinear mean-standard deviation framework for a competitive firm model under price risk. A utility function is proposed that nests various risk preference structures and risk neutrality as empirically refutable special cases. The empirical application using firm-level data finds evidence of decreasing absolute risk aversion, differences in the nature of relative risk aversion by firm size, and little support for the widely used linear mean-variance framework. The estimation results also show that ignoring risk and risk preferences can substantially overestimate output supply and input demand elasticities. (*JEL* D81)

I INTRODUCTION

Under uncertainty, individuals' economic decisions and their responses to price or income changes are significantly influenced by their risk attitudes. As a result, it is difficult to assess the merits and consequences of alternative economic incentives without information about the risk preferences of the targeted population. Also, in analytical uncertainty models, unambiguity in a decision maker's response to price or income changes is often secured by risk preference restrictions as in Sandmo [1970; 1971], Batra and Ullah [1974], and Pope [1980]. Whether such analytical results are meaningful depends, in part, on the validity of the preference restrictions on which they rest.

A sustained stream of applied research has focused on estimating risk preferences using economic data. Two distinct yet related strands of literature can be identified. The first infers agents' risk attitudes by testing restrictions on optimal decisions implied by alternative risk preference structures. For example, constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) imply that optimal choices are invariant to changes in initial wealth and in scale of wealth, respectively (Sandmo [1977]). A representative, though partial, list of such studies includes Cohn et al. [1975], Landskroner [1977], Siegel and Hoban [1982], Morin and Suarez [1983], Bellante and Saba [1986], Chavas and Holt [1990], and Pope and Just [1991]. The studies in the second stream attempt to directly estimate utility functional forms or risk aversion coefficients from data on individuals' choices. Some studies in this area are Friend and Blume [1975] Weins [1976], Hansen and Singleton [1983], Wolf and Pohlman [1983], Szpiro [1986], Love and Buccola [1991], and Saha et al. [1994].

This paper belongs to the second stream. It proposes a method that allows joint estimation of the degree or absence of risk aversion, structure of risk preference, and technology. Section II motivates the paper by arguing that the Arrow-Pratt measure of risk aversion imposes a priori restrictions on risk preference depending on the choice of the utility functional form. In section III, a flexible utility function in the nonlinear mean-standard deviation framework is proposed. The proposed form nests alternative risk preference structures as refutable special cases. Section IV applies the estimation method to a competitive firm model under price risk. Firm-level data from Kansas is used in the empirical analysis.

ABBREVIATIONS

NIARA Non-increasing absolute risk aversion
HARA Hyperbolic absolute risk aversion
EP. Expo-power utility function
MSU Mean-standard deviation utility function
CD Cobb-Douglas production function

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Section V contains estimation results. The findings suggest that Kansas producers' risk preferences are characterized by decreasing absolute risk aversion. While small producers exhibit increasing relative risk aversion, the null hypothesis of constant relative risk aversion is not refuted in the case of large producers. The findings provide little support for the widely used linear mean-variance framework. Evidence also suggests that ignoring risk and risk preferences can result in substantial overestimation of the supply and demand elasticities of a firm.

II ESTIMATION METHODS IN THE EXPECTED UTILITY FRAMEWORK

An advantage of testing behavioral restrictions implied by risk preference structures is that it does not require functional form assumptions about underlying utility. Often, however, empirical findings from studies in this approach preclude definite inference about risk preferences. Consider, for example, the comparative static result from the Sandmo [1971] model of a firm facing price risk: under nonincreasing absolute risk aversion (NIARA), optimal output is increasing in expected price. Observe, this preference restriction of NIARA is *sufficient*, but not necessary, for the result. Consequently, if estimation results show that output supply is indeed positively related to price, very little can actually be inferred about risk preferences. It is possible to conclude definitely that the sample individuals *do not* exhibit NIARA preferences only if estimation results show a negative relation between output supply and expected price.

This problem of ambiguity in inference obviously does not arise in studies that directly estimate a utility function. However, a feature common to studies in this stream is that risk parameter estimates are conditional. That is, the coefficient of absolute or relative risk aversion is estimated conditional upon a specific risk preference structure implied by the assumed functional form. For example, negative exponential utility—used by Love and Buccola and by studies employing the linear mean-variance approach such as Weins or Friend and Blume—imposes CARA and increasing relative risk aversion (IRRA). The power utility function (used by Hansen and Singleton, among others) imposes decreasing absolute risk aversion (DARA) and CRRA. Thus, commonly used utility functional forms allow data to reveal only the *degree* of risk aversion and not its *structure:* the latter is imposed by the utility functional form chosen. Furthermore, these functions, being intrinsically nonlinear in wealth, do not admit risk neutrality as a testable special case.

In this context, two relatively "flexible" functional forms warrant comments. The hyperbolic absolute risk aversion or HARA class of utility functions proposed by Merton [1971] and used by Wolf and Pohlman among others, has the flexibility to exhibit decreasing, constant or increasing relative risk aversion depending on parameter values. However, a utility function in the HARA family cannot exhibit CARA. In a recent article Saha [1993] proposed the expo-power (EP) utility function which has the flexibility to exhibit decreasing, constant, or increasing absolute risk aversion. But EP utility function is incapable of exhibiting CRRA under meaningful parameter values. Thus, even the relatively "flexible" functional forms impose a priori restrictions to the extent they preclude certain risk preference structures.

Because HARA and EP utilities are also intrinsically nonlinear, their parameter estimates can be used to test risk neutrality locally, but not globally. Consider, for example, the EP utility: $u(w) = 1 - \text{Exp}[-a \cdot w^b]$, where w denotes wealth, and a and b are parameters. The second derivative of EP utility has the same sign as $[a - a \cdot b \cdot w^b - 1]$; consequently, the parameter restriction implied by risk neutrality can be tested only at a particular level of w and not globally. Similar comments apply for the HARA class of utility functions. Although risk aversion is often accepted as a "stylized fact," its preclusion as a testable special case certainly limits the generality of a utility functional form.

A priori preclusion of certain risk preference structures by commonly used utility functional forms does not stem from their restrictive structures alone. It reflects a more general problem of the Arrow-Pratt measures of risk aversion within the expected utility approach. To elaborate, denote the Arrow-Pratt measures of absolute and relative risk aversion by $A(w) = -[u''(w)/u'(w)]$ and $R(w) =$

TABLE I
Alternative Risk Preference Configurations Using Arrow-Pratt Measures

	DRRA	CRRA	IRRA
DARA	feasible	feasible	feasible
CARA	not feasible	not feasible	feasible
IARA	not feasible	not feasible	feasible

$w \cdot A(w)$, where primes denote derivatives. Differentiation of $R(w)$ yields

$$(1) \qquad R'(w) = A(w) + w \cdot A'(w).$$

A number of restrictions on risk preference configurations follow immediately from the above equation. For example, if $u(w)$ exhibits CARA, i.e., $A'(w) = 0$, the $u(w)$ must exhibit increasing relative risk aversion, since $R'(w) = A(w) > 0$; CRRA or DRRA are precluded. Thus, in the Arrow-Pratt framework, irrespective of the degree of its flexibility, a utility function can either exhibit CARA or CRRA, never both. This explains the preference preclusion in HARA and EP utility functions noted above. Furthermore, it is evident from equation (1) that under CRRA or DRRA, since $A'(w) \leq -A(w)/w < 0$, the only compatible preference is DARA; CARA and IARA are both infeasible. These restrictions on risk preference configurations are summarized in Table I. Restrictions implied by Arrow-Pratt measures seem particularly severe in the face of ambiguous empirical evidence on the nature of risk aversion. Several empirical studies, including that of Wolf and Pohlman, have concluded that the hypotheses of "either increasing absolute or constant absolute risk aversion cannot be rejected" [1983, 847]. Evidence on the nature of relative risk aversion has also remained mixed. For example, Cohn et al., Szpiro, and Wolf and Pohlman report results in support of decreasing, constant and increasing relative risk aversion, respectively. To further complicate matters, analysts have argued that Arrow's justification of increasing relative risk aversion based on bounding conditions of the utility function "may be economically empty" (Friend and Blume [1975, 901]). It appears, therefore, neither empirical evidence nor analytical arguments are compelling enough to rule out configurations like CARA and DRRA or CARA and CRRA, though they are infeasible in the expected utility framework. Therefore, an alternative risk aversion measure is needed that can incorporate all possible risk preference configurations (i.e., all nine cells in Table I) as refutable special cases. This observation provides the central motivation for our study.

III RISK PREFERENCE STRUCTURES IN THE NONLINEAR MEAN-STANDARD DEVIATION FRAMEWORK

The nonlinear mean-standard deviation utility ($U(\mu, \sigma)$) framework provides flexibility in representing alternative risk preferences. The $U(\mu, \sigma)$ decision criterion hypothesizes that an agent's optimal choices are made by ranking alternatives through a preference function defined over the first two moments of random payoff, μ and σ.

To our knowledge, the $U(\mu, \sigma)$ decision criterion was first proposed by Fisher, in 1906. However, this decision framework gained prominence since the studies by Markowitz [1952] and Tobin [1958] and has been widely used in the analytical as well the empirical risk literature. Rosenzweig and Binswanger [1993] provide a recent empirical application. Sinn [1983] and Meyer [1987] have shown that neither normally distributed random payoff nor quadratic utility for agent's preference is necessary for the preference ordering under the expected utility and $U(\mu, \sigma)$ maximization approaches to be consistent. The consistency condition is met when the choice set is composed of random variables that belong to a "linear class" within which all distributions can be "transformed into one another merely by a shift and a proportional extension" (Sinn [1983, 56]). Equivalently, the consistency condition is satisfied when the "choice set [is]... composed of random variables which differ from one another only by location and scale parameters" (Meyer [1987, 422]).

TABLE II
Alternative Risk Preference Configurations Using MSU Utility

	DRRA	CRRA	IRRA
DARA	$\theta > 1, \theta > \gamma$	$\theta > 1, \theta = \gamma$	$\theta > 1, \theta < \gamma$
CARA	$\theta = 1, \theta > \gamma$	$\theta = 1, \theta = \gamma$	$\theta = 1, \theta < \gamma$
IARA	$\theta < 1, \theta > \gamma$	$\theta < 1, \theta = \gamma$	$\theta < 1, \theta < \gamma$

Note DARA (CARA, IARA) denotes decreasing (constant, increasing) absolute risk aversion. DRRA (CRRA, IRRA) denotes decreasing (constant, increasing) relative risk aversion.

Although the $U(\mu, \sigma)$ and expected utility approaches are analytically equivalent under the consistency condition, they differ significantly in their empirical tractability and their flexibility in representing alternative risk preferences. We proceed by formalizing an agent's risk attitude and choices in the $U(\mu, \sigma)$ framework.

Consider a decision maker whose utility function, U, is defined over the mean and the standard deviation of her random wealth, denoted by M and S; that is: $U(\mu, \sigma) \equiv U(M, S)$. Her risk attitude is reflected by

$$(2) \qquad A(M, S) \equiv -(U_S / U_M)$$

where subscripts denote partial derivatives. $A(M, S)$ is the slope of the indifference locus in the S-M space. Meyer has shown that, under the location and scale condition, various hypotheses concerning risk aversion measures in the expected utility setting can be translated into equivalent properties concerning $A(M, S)$. In particular:

1. risk aversion, neutrality and affinity correspond to $A(M, S) > 0$, $= 0$, and < 0, respectively;
2. the magnitude of $A(\cdot)$, when positive, reflects the degree of aversion to risk;
3. decreasing (constant, increasing) *absolute* risk aversion corresponds to $A_M < 0$ ($= 0$, > 0);
4. decreasing (constant, increasing) *relative* risk aversion is reflected by $A_t(tM, tS) < 0$ ($= 0$, > 0), respectively, for $t > 0$

The foregoing restrictions provide the basis for proposing the following flexible utility function:

$$(3) \qquad U(M, S) = M^{\theta} - S^{\gamma}$$

where θ and γ are parameters and it is assumed throughout that $\theta > 0$. We will call this the mean-standard deviation utility function or MSU. Under MSU, the risk attitude measure, A, is given by

$$(4) \quad A(M, S) \equiv -(U_S / U_M) \equiv (\gamma / \theta)\, M^{1-\theta} S^{\gamma-1}.$$

It can be verified that MSU exhibits

1. risk aversion, neutrality, and affinity as $\gamma > 0$, $= 0$, and < 0;
2. decreasing, constant, and increasing *absolute* risk aversion as $\theta > 1$, $= 1$, and < 1; and
3. decreasing, constant, and increasing *relative* risk aversion as $\theta > \gamma$, $\theta = \gamma$, and $\theta < \gamma$.

The various risk preference configurations under MSU are summarized in Table II. MSU's flexibility is evident from the comparison of Tables I and II. Also note that MSU embeds the widely used linear mean-standard deviation model as a refutable special case wherein $\gamma = 1$ and $\theta = 1$.

IV. ESTIMATION OF TECHNOLOGY AND PREFERENCES UNDER MSU

In this section we develop an empirical framework for the joint estimation of a firm's risk preference and production technology using MSU. Consider a single-product competitive firm's decision problem under price risk. The firm's random wealth, $\tilde{W}$, is defined as follows:

$$(5) \qquad \tilde{W} = \tilde{p} \cdot Q - C(\mathbf{r}, Q) + w$$

where $\tilde{p}$ is the random output price, Q denotes output, C denotes the cost function, $\mathbf{r}$ is a vector of input prices, and w is exogenous wealth (initial endowment). The wealth structure defined in (5) implies that "all random alternatives available to the firm are positive linear transformations of the given random variable [$\tilde{p}$] and hence are related to one another by location and scale parameters" (Meyer [1987, 427]). That is, consistency between expected utility and $U(\mu, \sigma)$ maximization holds in the competitive firm model under price risk. Observe that nothing has been assumed about the distribution of random price. The consistency condition is satisfied solely by the fact that the firm's wealth function is linear in $\tilde{p}$.

Assume that the firm observes past realizations of random price to form its perception about the mean and standard deviation of the price distribution, denoted by $\bar{p}$ and σ_p. The firm's opportunity set is described by

$$M = \bar{p} \cdot Q - C(\mathbf{r}, Q) + w$$

$$S = \sigma_p \cdot Q.$$

In the $U(\mu, \sigma)$ decision framework the firm's output choice problem is

$$\text{(6)} \qquad \max_{Q} \; U(M, S) \equiv U(\bar{p} \cdot Q - C(\mathbf{r}, Q) + w, \sigma_p \cdot Q).$$

Under the assumption that the producer's utility function is given by MSU, the first-order condition of the problem in equation (6) can be written as

$$\text{(7)} \qquad \bar{p} - C_Q(\mathbf{r}, Q) = (\gamma / \theta) M^{1-\theta} Q^{\gamma-1} (\sigma_p)^{\gamma}.$$

If the firm is risk neutral, i.e., $\gamma = 0$, the right-hand side of equation (7) is zero and the above first-order condition simply equates expected output price to marginal cost, $C_Q(\cdot)$. Taking logarithms, equation (7) can be rewritten in implicit estimation form as

$$\text{(8)} \qquad \ln [\bar{p}_i - C_{Qi} (\mathbf{r}_i, Q_i, \beta)] - \ln (\gamma / \theta) - \gamma \ln (\sigma_{pi}) + (\theta - 1) \ln M_i + (1 - \gamma) \ln Q_i + \varepsilon_i = 0.$$

The subscript i corresponds to the ith observation, ε_i denotes error in optimization and β is the set of technology parameters embedded in the cost function whose specific form is assumed to be known through prior estimation. For efficiency gain, equation (8) can be estimated with the cost and/or production functions as a system of equations with correlated errors. The estimation will provide joint estimates of the technology and utility function parameters which we denote by the vector $\Omega \equiv \{\beta, \theta, \gamma\}$. The estimation equation (8) is quite general and can be used for any underlying technology by substituting the appropriate marginal cost term, C_Q.

Furthermore, estimation of equation (8) is fairly straightforward and does not pose onerous problems. In contrast, in the expected utility framework, unless a simple discrete probability density is assumed for the distribution of random wealth or a linear mean-variance framework is posited, the first-order conditions typically involve integrals. Consequently, estimation requires computer software that can perform numeric integration within a numeric optimization routine as in Saha et al. [1994]. Estimation of MSU-based equations as in equation (8) is considerably simpler. In fact, all estimations in this paper were undertaken using SHAZAM, a widely used econometrics software package.

The Data Set

The firm-level data used in this study come from the computerized farm accounting records of the Farm Management Data Bank at the Department of Agricultural Economics, Kansas State University (Langemeier [1990]). The producers were selected from the Data Bank by the criterion that in each year at least 95% of row crop acreage was devoted to wheat. Consequently, the selected producers were predominantly single-product firms. The data set comprised 60 observations on 15 producers for the period 1979 to 1982.

Data included expenditures incurred and/or estimates of rental values for a wide array of inputs and crop output quantities. The data also included government payments received and nonfarm asset income. Because prices paid for inputs were not available from the firm-level data set, Kansas state-level price data, compiled by Robert Evenson and his associates at Yale University, were used as proxies for input prices. Major sources of their data were the U.S. Department of Agriculture's *Agricultural Statistics*, *Agricul-*

TABLE III
Summary Statistics on Kansas Firms

	Variable Mean (standard deviation)		
	All Firms	Small Firms	Large Firms
Capital Inputs: x_1	44.051 (23 028)	32.017 (9 8392)	52 073 (25.784)
Material Inputs x_2	70 059 (28 435)	43.206 (17.736)	87 961 (18 226)
Capital Inputs Price r_1	9070 (0 0712)	9143 (0 0665)	.9022 (0 0746)
Material Inputs Price. r_2	9383 (0743)	.9389 (0 0769)	.9379 (0736)
Wealth Endowment w	52 396 (17 195)	42 760 (5 4848)	58 820 (19 301)
Output· Q	132 06 (56.355)	78.071 (25.582)	168.05 (39.903)
Expected Output Price· $\bar{p}$	1 1748 (0 0429)	1 1748 (0 0429)	1.1748 (0.0429)
Standard Deviation of Output Price σ_p	0.4648 (0 1058)	0 4648 (0 1058)	0 4648 (0 1058)

tural Prices, *Field Crops Production Disposition and Value*, and *State Farm Income and Balance Sheet Statistics*, and the Chicago Board of Trade's *Statistical Annual.*

The Kansas wheat producers were assumed to use two inputs: capital (x_1) and materials (x_2). Summary statistics for the data used in estimation are presented in Table III. Expenditures on capital inputs, x_1, included an interest charge on land and building equity, cash farm rent, building and machinery depreciation, and real estate taxes. The materials category, x_2, included machinery and machinery hire, fertilizer, pesticides, seed, and miscellaneous cash expenses. All price aggregates were computed as geometric means using expenditure or revenue shares as the weights. Nonfarm asset income was used as a proxy for producers' wealth endowment, w. The data on the mean of wealth M was generated by adding expected farm profits to nonfarm asset income.

In addition to the data set on all firms, two additional data sets corresponding to small and large producers were created. To create these data subsets, all firms were first ranked according to average annual output (the ranking on the basis of average expected wealth, M, was identical). The top nine firms were then categorized as large and the remaining six constituted the small category. The summary statistics for the two groups of producers are reported in Table II.

Estimation Details

Generation of data on the mean and standard deviation of price moments, $\bar{p}$ and σ_p, warrants a few comments. Observe that the optimization problem in equation (6) was set out in a "timeless" framework since it is essentially a static problem and, therefore, the same in each period. Furthermore, it was assumed that the firm's optimal output choice in each period is based on subjectively formed price moments. Clearly, data on the moments cannot be collected directly. Therefore, a Nerlovian "quasi-rational expectations" approach was adopted. Nerlove et al. [1979] proposed that forecasts from an ARIMA (autoregressive integrated moving average) process can be used to generate data on expectations.

A series of tests based on the method of Box and Jenkins [1976] were performed using annual data on Kansas state-level wheat prices for the period 1955 to 1982. The autocorrelation function of the price series showed gradual decay, while the partial autocorrelation function indicated a clear "cutoff" after the second lag, typical of an AR(2) process. Based on these diagnostics, the Kansas wheat price process was assumed to have the following structure:

$$(9)\qquad p_t = \varphi_0 + \varphi_1 p_{t-1} + \varphi_2 p_{t-2} + e_t$$

where $e_t \sim (0, \sigma_e^2)$, for all $t = 1,\ldots T$.

In the AR(2) model above a producer's forecast of the conditional mean price at period t (i.e., conditional upon the information available up to period t–1) is given by

$$(10a)\quad \mathrm{E}[p_t \mid \Gamma_{t-1}] \equiv \bar{p}_t = \varphi_0 + \varphi_1 p_{t-1} + \varphi_2 p_{t-2}$$

where Γ_{t-1} denotes the information set at period t–1. If nothing more is assumed about the random variable p_t in equation (9), then its conditional variance is simply σ_e^2 which is a time-invariant constant. Thus, an AR specification alone is not adequate to model producers' forecast of price variance.

Following Engle [1982], we use an autoregressive conditional heteroskedastic (ARCH) framework to model the time-varying variance forecasts where changes in forecasts are induced by past information. In particular:

$$(10b)\quad Var\,[\tilde{p}_t \mid \Gamma_{t-1}] \equiv \sigma_{p_t}^2 = \exp(a_0 + a_1 e_{t-1}^2)$$
$$= \exp[a_0 + a_1 \cdot (p_{t-1} - \varphi_0 - \varphi_1 p_{t-2} - \varphi_2 p_{t-3})^2]$$

where equation (9) has been used to substitute for e_{t-1} in equation (10b). The specification in equation (10b) corresponds to an exponential ARCH(1) model discussed in Harvey [1993, 275–80].[1] The exponential ARCH or EARCH, proposed by Nelson [1991], has the advantage of ensuring that $\sigma_{p_t}^2$ is strictly positive for all finite parameter values and price realizations.

Assuming Gaussian errors,[2] maximization of the following log likelihood function (with omitted constants)

$$(10c)\quad -\frac{1}{2}\sum_{t=1}^{T}\{a_0 + a_1(p_{t-1} - \varphi_0 - \varphi_1 p_{t-2} - \varphi_2 p_{t-3})^2 + \{(p_t - \varphi_0 - \varphi_1 p_{t-1} - \varphi_2 p_{t-2})^2 / \exp[a_0 + a_1(p_{t-1} - \varphi_0 - \varphi_1 p_{t-2} - \varphi_2 p_{t-3})^2]\}$$

provides the maximum likelihood (ML) estimates of the parameters φ_0, φ_1, φ_2, a_0, and a_1. Estimation was undertaken using Kansas state-level wheat price data for the period 1955 to 1982. The ML estimates of the parameters in equation (10c) are reported in Table IV. Using these parameter estimates and the expressions in equations (10a) and (10b), data on price moments for 1979–1982—the four-year period for which data on other variables are available—were generated.

The underlying assumption of the foregoing process of generating data on price moments is that, in each period, the decision maker observes past price realizations to form perceptions about price moments, which are then taken as parameters in the optimal output choice problem. In other words, the price moment formation is exogenous to the decision problem in equation (6). Furthermore, since state-level time series on wheat prices were used in estimating the parameters in equation (10c), all producers in the sample were assumed to face the same price.

To determine the technology structure of sample producers, the translog, generalized Leontief, quadratic and Cobb-Douglas production functions were estimated using the firm-level data. The value of the estimated log likelihood functions, with appropriate Jacobian correction, and the number of parameters for each functional form are presented in Table V.[3] Using Pollack and Wales's [1991]

1. Higher order ARCH models were also estimated. But these models were dominated by the ARCH(1) specification under standard model selection criteria

2. Test for normality of the price distribution was conducted using Kansas state-level prices for 1955 through 1982 The test procedure is described in Kiefer and Salmon [1983] The Wald χ^2 test statistic (for the null hypothesis of normality) was 3 454 (P-value = 0 179), not rejecting the null

3 The dependent variable in the translog and Cobb-Douglas specifications is log of output, $\log(Q_i)$, whereas in the quadratic and generalized Leontief specifications it is output, Q_i. Therefore, the Jacobian term, $\sum \log(1/Q_i)$, was added to the translog and Cobb-Douglas log likelihood values, ensuring comparability across specifications.

TABLE IV
ML Estimates of Parameters of the Price Distribution

	Parameter Estimate (standard error)
Mean Parameter*	
φ_0	0.8579
	(0 2268)
φ_1	0.9121
	(0.1102)
φ_2	–0 2334
	(0.1290)
Variance Parameter**	
a_0	–1 1572
	(0.2858)
a_1	–0 9452
	(0 6964)

*See equation (10a) in text.
**See equation (10b) in text

likelihood dominance criterion for testing non-nested hypotheses, the Cobb-Douglas (CD) was found to dominate the quadratic and generalized Leontief functions. The same conclusion followed from Akaike's information criterion of model selection. The Translog nests the CD as a special case. The hypothesis that all the higher order terms in the Translog function are jointly equal to zero was not rejected, suggesting again that CD is the appropriate form. The following are CD production and cost function estimation equations:

$$\text{(11a)} \quad \ln Q_{it} = \beta_0 + \beta_1 \ln x_{1_{it}} + \beta_2 \ln x_{2_{it}} + \varepsilon_{1it}$$

$$\text{(11b)} \quad \ln C_{it} = \ln K + 1/(\beta_1 + \beta_2)$$
$$[\beta_1 \ln r_{1it} + \beta_2 \ln r_{2it} + \ln Q_{it}] + \varepsilon_{2it}$$

where

$$K = \beta_0^{-(1/\beta_1 + \beta_2)} [(\beta_1/\beta_2)^{(\beta_2/\beta_1 + \beta_2)}$$
$$+ (\beta_2/\beta_1)^{(\beta_1/\beta_1 + \beta_2)}]$$

r_k is the price of the kth input, denoted by x_k, $k = 1,2$, and subscripts i and t denote the ith producer and the tth time period, respectively. Marginal cost structure corresponding to CD technology was substituted for the term C_Q in equation (8) to yield the following estimation equation:

$$\text{(11c)} \quad 0 = \ln[\bar{p}_t - (K/\beta_1+\beta_2)(r_{1it}^{\beta_1} r_{2it}^{\beta_2})^{(1/\beta_1+\beta_2)}$$
$$Q_{it}^{(1-\beta_1-\beta_2/\beta_1+\beta_2)}]$$
$$- \ln(\gamma/\theta) - \gamma \ln\sigma_{pt} + (\theta - 1)\ln M_t$$
$$+ (1-\gamma)\ln Q_{it} + \varepsilon_{it}$$

where 0 on the left-hand side of equation (11c) denotes a vector of zeros. The three equations, (11a), (11b) and (11c), were jointly estimated as system of nonlinear equations, with correlated errors. Since equations (11b) and (11c) contain endogenous elements, the nonlinear three stage least squares (3SLS) procedure was adopted in estimating equations (11a) through (11c), with the input prices and the price moments being the exogenous variables in the system.

V ESTIMATION RESULTS

The nonlinear 3SLS estimates of production technology (β_0, β_1, β_2) and utility function (θ, γ) parameters for all firms and for the two categories are presented in Table VI. All parameter estimates are significant at the 1% level. In particular, observe that the estimate of the utility function parameter, $\hat{\gamma}$, is highly

TABLE V
Estimation of Alternative Production Technologies

Production Technology	Number of Parameters	Value of Log Likelihood Function*
Cobb-Douglas (CD)	3	–287 012
Quadratic	6	–289.579
Translog (TL)	6	–285.106
Generalized Leontief	6	–289 605
F-test statistics for TL vs CD** (P-value)		1.1807 (0.3257)

*Appropriate Jacobian adjustment has been made in computing the log likelihoods. For further details see footnote 3 in text.

**The null hypothesis is that the coefficients of $\ln x_1^2$, $\ln x_2^2$ and $\ln x_1 x_2$ are jointly equal to zero.

significant in each case, clearly rejecting the null hypothesis of risk neutrality. The measure of risk aversion, $A \equiv -U_S/U_M$, evaluated at sample means, shows that the degree of risk aversion does differ by producer category and its pattern is consistent with decreasing absolute risk aversion. Not unexpectedly, the null hypothesis of CARA preferences is refuted for all firms and for the two categories in favor of DARA. Also, the null hypothesis of a linear mean-standard deviation model, H_0: $\theta = \gamma = 1$, is clearly rejected in favor of a nonlinear specification for all three categories.

The finding on the nature of relative risk aversion is less homogeneous. In the case of small and all producers, $(\hat{\gamma} - \hat{\theta})$ is positive and statistically significant at the 1% level, suggesting a preference structure characterized by increasing relative risk aversion. However, for large producers $(\hat{\gamma} - \hat{\theta})$ is negative and the null hypothesis of constant relative risk aversion, H_0: $(\gamma - \theta) = 0$, is not rejected. This finding that risk preferences vary by firm size underscores the importance of a flexible utility structure. Under EP utility, for example, this result would have been precluded since EP does not admit constant relative risk aversion.

Our findings on relative risk aversion are, in the main, consistent with the results in previous studies. It may be recalled from the discussion in section I that the evidence on the nature of relative risk aversion is mixed. Findings supporting decreasing (Cohn et al.), constant (Szpiro), and increasing (Wolf and Pohlman) risk aversion have been reported. It should be noted, however, that most prior studies have not investigated whether the nature of relative risk aversion preference differs according to wealth levels.

The estimate of $A(M, S)$ furnishes an additional piece of information that does not have its direct counterpart in the expected utility framework. Under MSU, the derivative of $A(\cdot)$ with respect to S has the same sign as $\gamma(\gamma - 1)$. Under $\gamma > 0$, $A_S(\cdot)$ conveys how aversion to risk changes as the wealth distribution becomes more risky. It is evident from Table VI that for all three categories γ is significantly greater than one and, therefore, $A_S(\cdot)$ is positive. This accords with intuition since individuals' aversion to risk is likely to increase with increased volatility of wealth.

Turning now to technology parameters, observe that estimates suggest returns to scale of 0.9238, 1.1961, and 0.9599 for all, small and large firms, respectively. In each case constant returns to scale is rejected at the 1% level of significance. Also, the technology parameters are highly significant for all three categories.

Table VII contains the estimates and the standard errors of output supply and input demand elasticities for all producers. The computation method for these elasticities warrants a few comments. Even under as simple a production technology as the CD, the first-order conditions in equation (7) cannot be solved explicitly for optimal output level Q^*. The absence of a closed form solution precludes direct estimation of the output supply function to recover the supply elasticities with respect

TABLE VI
Parameter Estimates and Test Results

Parameter/Test	Explanation	All	Firm Category Small	Large
β_0		3 1434 (0 0566)	1 0733 (0.0153)	2.7967 (0 0333)
β_1	Production Function Parameters*	0.3071 (0.0257)	0 5770 (0.0277)	0.2726 (0 0175)
β_2		0 6167 (0.0264)	0.6191 (0.0239)	0.6873 (0 020)
θ	Utility Function	3 1397 (0 0553)	1 8645 (0.0435)	3.8424 (0 0976)
γ	Parameters*	3.2775 (0 0490)	1.9564 (0 0452)	3.7945 (0.0939)
A	Measure of absolute risk aversion ($\equiv -U_S/U_M$) evaluated at sample mean*	0.6384 (0 1383)	0 8966 (0 1661)	0.5308 (0.2414)
H_0. $\beta_1 + \beta_2 = 1$	Constant returns to scale**	−15.3560 (0 000)	20 9663 (0 000)	−7.3814 (0 000)
H_0: $\theta = \gamma = 1$	Linear $U(M, S)$ model***	318 5667 (0 000)	42 8321 (0.000)	108.1131 (0.000)
H_0. $\theta = 1$	CARA Preferences****	38.6676 (0 000)	19.8580 (0 000)	29 1257 (0 000)
H_0. $(\gamma - \theta) = 0$	CRRA Preferences*****	2 8123 (0 005)	2 0355 (0 042)	0.4867 (0 6264)

Note
*Standard errors in parentheses
**Asymptotic t-statistics, P-value in parentheses
***Asymptotic $\chi^2(2)$ test statistic, P-value in parentheses
****Constant Absolute Risk Aversion, asymptotic t-statistics, P-value in parentheses
*****Constant Relative Risk Aversion, asymptotic t-statistics, P-value in parentheses

to $\bar{p}$, **r**, or w. However, since the functional forms as well as the parameters of the utility and production functions have been estimated, the elasticities can be directly computed from the analytical comparative static expressions. For example, optimal output response to expected price is

$$(12) \qquad Q_{\bar{p}}^{*}(\bar{p}, \mathbf{r}, w; \hat{\Omega}) \equiv \{U_M + Q^{*} [\,U_{MM}(\bar{p} - C_Q)]\}\,/-D$$

where

$$D \equiv [\,U_{MM} \cdot (\bar{p} - C_Q)^2 + U_{SS} - C_{QQ}U_M\,] < 0$$

by the second-order sufficient condition of equation (6). All terms in the right-hand side of equation (12) are known functions of the parameter estimates $\hat{\Omega} \equiv (\hat{\beta}, \hat{\theta}, \hat{\gamma})$ and data variables. For example, $U_M \equiv \theta M^{\theta-1}$, $U_S \equiv \gamma S^{\gamma-1}$, etc. Thus, one can directly compute $Q_{\bar{p}}^{*}$ and hence the supply elasticity $\varepsilon_{\bar{p}}^{Q}$. The standard error of the elasticity can then be calculated from $V(\hat{\Omega})$ using the delta method, since $Q_{\bar{p}}^{*}$ is a nonlinear function of $\hat{\Omega}$.[4] This analytic expression-based method was also adopted for computing the price elasticities for inputs. The output supply and input demand elasticity estimates (evaluated at the sample means) and their standard errors are reported in Table VII. All elasticities have the expected signs and, with few exceptions, are significant at the 1% level.

4 If $F[\Omega]$ denotes a function of the parameter vector Ω, the variance-covariance matrix of $F[\Omega]$ is approximated by $G^T V[\Omega] G$, where $G \equiv \partial F/\partial W$, $V[\Omega]$ is the variance-covariance matrix of Ω, and ^{T} denotes transpose

TABLE VII
Demand and Supply Elasticity Estimates (All Firms)

Elasticity with Respect to:		Output Q	Elasticity of: (standard errors) Capital Input x_1	Materials Input x_2
$\bar{p}$	expected output price	1 0976 (0.0708)	1 1772 (0.1816)	1 1936 (0.1111)
r_1	price of x_1	–0 2857 (0.0464)	–1 3713 (0 3116)	0 2197 (0 1225)
r_2	price of x_2	–0 4770 (0.0464)	0.3614 (0 2015)	–0.9536 (0 1573)
w	wealth endowment	0.2776 (0.0015)	0 2955 (0 0183)	0 2712 (0 0371)

To understand the inferential consequences of ignoring risk effects, the first-order condition of profit maximization under certainty (equating output price to marginal cost), the CD production and cost functions were estimated as a system of equations using the same data set. Using the certainty model parameter estimates, output supply and input demand elasticities were computed. Note, under CD technology in a risk-free setting the output supply and input demand elasticity estimates follow directly from the production function parameter estimates. For example, output supply elasticity in a risk free setting is $(\beta_1 + \beta_2)/(1 - \beta_1 - \beta_2)$. These estimates are presented in Table VIII.

The returns to scale estimates under the two settings show a statistically significant difference (the t-statistic is 4.2463). More importantly, it is evident from the comparison of output supply and own-price input demand elasticities that ignoring risk effects yields a substantially more elastic price response. The differences in elasticity estimates are statistically significant at least at the 5% level. This was found to be true also for the cross-price elasticities (which are not reported in the interest of brevity). In models of uncertainty, any parameter change has two effects on optimal choices: a direct or "substitution" effect and an indirect or "income" effect attributable to the change in wealth induced by the parameter change as discussed, for example, in Chavas and Pope [1985]. The latter effect is absent under certainty, risk neutrality or constant absolute risk aversion. Since, in the empirical application, risk neutrality and constant absolute risk aversion were clearly rejected, the difference in the elasticity estimates under the two settings stems, in part, from the r_1, r_2, and $\bar{p}$ change-induced wealth effects that are ignored in the risk-free setting. The elasticity difference is also explained by the difference in the estimates of technology parameters, β_0, β_1, and β_2, under the two settings.

Observe in Table VIII that the estimated standard errors for all three technology parameters from the uncertainty model are about 50% smaller than their counterparts in the risk-free setting. This suggests that the joint estimation of technology and utility function parameters may be more efficient than the separate estimation of technology.

VI. CONCLUDING COMMENTS

I have proposed a method for jointly estimating the degree or absence of risk aversion, structure of risk preference, and production technology in the nonlinear mean-standard deviation approach. I argue that the Arrow-Pratt measures of risk aversion and commonly used utility functional forms, including those in the HARA family, impose a priori restrictions on risk preference. As an alternative, I propose a flexible utility function in the nonlinear mean-standard deviation approach. This utility function nests alternative risk preference structures, including risk neutrality, as refutable special cases. The method was applied to a competitive firm model under price risk. Em-

TABLE VIII
Estimates of Technology Parameter under Price Risk and Risk-free Settings (All Firms)

Parameter/Test	Joint Estimation under Certainty	Joint Estimation under Price Risk
	(standard errors in parentheses)	
β_0	3.3079	3 1434
	(0.1254)	(0 0566)
β_1	0 2169	0 3071
	(0.0406)	(0.0257)
β_2	0 6672	0 6167
	(0 0357)	(0 0264)
Returns to scale ($\beta_1 + \beta_2$)	0 8841	0 9238
	(0 0079)	(0 0050)
ε_p^Q. supply elasticity	7 6295	1 0976
	(0 5849)	(0 0708)
ε_{11}. own price elasticity of capital input	–2 8720	–1 3713
	(0.4463)	(0.3116)
ε_{22}. own price elasticity of materials input	–6 7575	–0 9536
	(0 3383)	(0 1573)

pirical estimation used farm-level data from Kansas. The findings suggest that Kansas producers' risk preferences are characterized by decreasing absolute risk aversion and that the nature of relative risk aversion varies by size category. While small firms are found to exhibit increasing relative risk aversion, the null hypothesis of constant relative risk aversion is not refuted in the case of large firms. Despite its ubiquity in applied research, the linear mean-variance framework is also unambiguously refuted.

Finally, evidence suggests that ignoring risk and risk preference can result in substantially overestimated supply and demand elasticities. I hasten to add, however, that given the relatively small sample size, caution is warranted in drawing general conclusions from these findings. Further research, especially with larger data sets, can ascertain whether the same conclusion holds for firms in other industries and for other technologies I believe that the main contributions of the present study lie not in its findings but in suggesting a flexible utility function and in proposing a general, yet tractable, empirical framework for the joint estimation of technology and risk preferences. It is hoped that the proposed utility function and estimation procedure will find other applications in emerging areas of the risk literature. In particular, extending the analytical and the empirical model to the multi-output firm setting may provide further insights.

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