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Refutable implications of the firm model under risk

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Curvature properties of the indirect utility function are shown to be necessary and sufficient for refutable behavioural postulates in the form of comparative static results, reciprocity relations, and restrictions on output and input responses for firm models under risk. These postulates are independent of onerous restrictions on risk preference, technology, or random variable distribution. As an exposition, the complete set of refutable implications for a multi-output firm operating under price risk is derived. At the data means, these implications are not rejected when they are tested using firm-level data, but both risk neutrality and constant absolute risk aversion are rejected. At individual observations, one implication is rejected almost as frequently as risk neutrality and constant absolute risk aversion.

I. INTRODUCTION

Karl Popper's seminal treatise on falsifiability and the logic of scientific analysis was published in 1934. Since then, leading theorists, including Samuelson (1947), Kuznets (1963), Blaug (1980), and Silberberg (1990), have argued that the paradigm of economics, to be useful, must consist of refutable propositions. Silberberg's views on this issue are unequivocal and cogently expressed. He writes:

> What types of theories are useful in empirical science, then? The only theories that are useful are those that might be wrong, i.e., might be refuted, but are not refuted.

He proceeds to explain:

> The testing of a theory usually involves two fairly distinct processes. First, the purely logical aspects of the theory are drawn out. That is, it is shown that the behavioral postulates imply certain behavior for the variables of the theory. Then, at a later stage, the theoretical constructs are applied to real data, and the theory is tested empirically (Sillberberg, 1990, pp. 13–15).

This paper deals with both of these processes of testing a theory in the context of firm decisions under risk. Its objectives are (a) to derive the complete set of refutable propositions for the competitive firm model under a general wealth structure that encompasses price and output risk as special cases and (b) to empirically test these propositions using firm-level data.

In the framework of certainty, the refutable implications of the competitive firm model are well known. For example, the signs of the derivatives of a profit function and the symmetry and positive semidefiniteness of the matrix of its second derivatives contain all refutable implications of a competitive firm model under certainty. A large body of applied production studies have empirically tested some or all behavioural postulates implied by these profit function properties (see Shumway, 1995, and Fox and Kivanda, 1994, for surveys of this literature).

In the literature on firm models under uncertainty, two significant areas of deficiency become evident. First, the properties of the dual function, that is, the indirect utility function and the behavioural postulates these properties imply have not been laid out systematically. Partly as a consequence of this inadequacy, some authors have concluded that '... there are no refutable hypotheses that follow from the postulate of expected utility maximization. To a Popperian, interesting as modern risk models may be, their development does not constitute scientific research' (Fox and Kivanda, 1994, p. 9). We contend that such disparagement of the expected utility maximization postulate is unwarranted. Several insightful studies in the production literature have explicitly set out empirically refutable comparative static results that follow from the competitive firm model under risk (see, for example, Chavas and Pope, 1985; Paris, 1988; 1989; Paris *et al.*, 1993). In fact, Paris *et al.* (1993) deal explicitly with the task of formulating refutable implications of economic models under risk. The incompleteness of this

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stream of literature, therefore, does not lie in its failure to specify refutable implications. What remains less developed are the properties of the dual, i.e. the indirect utility function. In particular, the dual's curvature properties and the systemic relation between such properties and empirically refutable comparative static and reciprocity results have remained unexplored.

The second deficiency of firm studies under risk is the absence of empirical studies that have attempted to test the refutable behavioural postulates. Although several of the cited papers have specified refutable comparative static and reciprocity results for firm models under risk, we did not find a single study that empirically tested such results.

This research attempts to address both of these deficiencies. The complete set of indirect utility function properties for a competitive firm under risk is developed in Section II. It is demonstrated that the indirect utility function's curvature is both necessary and sufficient for refutable behavioural postulates in the form of comparative static results, reciprocity relations, and restrictions on output and input responses. Importantly, these postulates are free from risk preference restrictions – they hold under any risk preference structure, including risk neutrality. Further, the postulates do not rely on any distributional restrictions on random variables or on technology assumptions. All that is needed is the existence of a solution to the firm's expected utility maximization problem. In Section III, as an exposition of the paper's analytical results, the refutable implications for a multioutput firm operating under price risk are derived. In Section IV, an estimation framework for testing the complete set of refutable implications for a firm model under price risk is proposed and applied to firm-level data. The behavioural postulates are supported by the data.

II. INDIRECT UTILITY FUNCTION PROPERTIES

Following Feder (1977), we consider a random wealth structure that is common to many firm models under risk:[1]

$$\tilde{W} \equiv M(\mathbf{x}; \beta, \cdot) + S(\mathbf{x}; \tilde{\varepsilon}; \cdot) + I \tag{1}$$

where $\tilde{W}$ denotes random wealth, $\mathbf{x} \in R^n$ is an $n \times 1$ vector of decision variables, $\tilde{\varepsilon}$ is a vector of random variables, β is a parameter vector, I is exogenous income, and $\cdot$ denotes the additional parameters that have been suppressed in the twice-differentiable functions M and S. We also assume:

$$E[S(\mathbf{x}; \tilde{\varepsilon}; \cdot)] = 0 \tag{2}$$

where E denotes the expectation operator. (2) implies:

$$\bar{W} \equiv E[\tilde{W}] = M(\mathbf{x}; \beta, \cdot) + I \tag{3}$$

Observe that the parameter vector, β, enters only the non-random part of wealth, M, but not the random part, S. The wealth structure implied by (1) and (2) is quite general. For example, in firm models under output risk with Just-Pope type production functions,[2] the wealth function clearly satisfies (1) and (2) (Pope and Kramer, 1979). In firm models of output choice under price risk (Sandmo, 1971), x is a scalar denoting output, and $\tilde{\varepsilon} = \bar{\varepsilon} + \tilde{e}$ is random price with mean $\bar{\varepsilon}$. In these models, mean wealth $\bar{W}$ is given by: $\bar{\varepsilon} \cdot x - c(x, \mathbf{r}) + I$, where $c(\cdot)$ denotes the cost function and $\mathbf{r}$ is a vector of input prices. Thus, $\bar{\varepsilon} \cdot x - c(x, \mathbf{r})$ corresponds to the function M in (1) with $\beta \equiv \{\bar{\varepsilon}, \mathbf{r}\}$ being the vector of parameters within M. Similar separation of the wealth structure holds in input choice models under price risk (Batra and Ullah, 1974; Pope, 1980). The reader may verify that the wealth structure in (1) and (2) is common to many other firm models including Batra (1974, 1975), Feder *et al.* (1980), Holthausen (1979), Just and Zilberman (1986), Chavas (1987), and Paris (1989).

A competitive firm, whose wealth is defined in (1), has the following indirect utility function:

$$V(\beta, I, \cdot) \equiv \operatorname*{Max}_{x} \{E[u(M(\mathbf{x}, \beta, \cdot) + S(\mathbf{x}; \tilde{\varepsilon}; \cdot) + I)]\} \tag{4}$$

where u denotes the firm's utility function that is increasing in wealth. Let $\mathbf{x}^*(\beta, I, \cdot)$ denote the optimal vector of decision variables. The paper's analytical results are summarized below.

Proposition 1. The indirect utility function defined in (4) has the following properties:
(i) Increasing (decreasing) in β if M is increasing (decreasing) in β. Increasing in I.
(ii) Quasiconvex in β and I if M is convex in β.

Proposition 2. V quasiconvex in β and $I \Leftrightarrow \Omega$ symmetric and positive semidefinite (SPSD), where $\Omega \equiv M_{\beta\beta} + M_{\beta x}\{\mathbf{x}^*_\beta - \mathbf{x}^*_I M_\beta\}$.

Corollary. Under risk neutrality or constant absolute risk aversion, $\mathbf{x}^*_I = 0$, and M convex in $\beta \Leftrightarrow M_{\beta\beta} + M_{\beta x}\mathbf{x}^*_\beta$ SPSD.

The proof of the two propositions and the corollary are presented in the Appendix. These results are further clarified

[1]The following notation will be used throughout the paper: $\mathbf{R}^n_+$ denotes the non-negative orthant in the n-dimensional real space. If $h(\mathbf{x}, \mathbf{y})$ is a real-valued function of two vectors $\mathbf{x}$ and $\mathbf{y}$, then $h_\mathbf{x}$ denotes the vector of partial derivatives of $h(\cdot)$ with respect to $\mathbf{x}$, and $h_{\mathbf{xy}}$ the matrix whose ijth element is $\partial^2 h(\cdot)/\partial x_i \partial y_j$. Transpose notation for vectors or matrices is not used. If $\mathbf{x}$ denotes an $n \times 1$ vector then $\mathbf{x} > 0$ implies $x_i > 0$ for all $i = 1, \ldots, n$, where i denotes the ith element.
[2]The Just–Pope production function is given by $Q = h(\mathbf{x}) + g(\mathbf{x})\tilde{\varepsilon}$, where Q is output, $\mathbf{x}$ is a vector of inputs, h and g are functions, and $\mathbf{E}[\tilde{\varepsilon}] = 0$ (Just and Pope, 1978).

through an application presented in the next section. Here we will limit our discussion to a few comments.

First, the results in Proposition 1(i) can be viewed as first-order curvature properties of the indirect utility function. Proposition 1(ii) is a fundamental result on the second-order curvature property of the indirect utility function, and it constitutes the counterpart to convexity of the profit function for a firm operating under certainty. Importantly, it is this property that implies and is implied by the refutable qualitative results contained in Proposition 2. Second, in most applications (in fact, in all firm models of risk cited earlier), the wealth structure is such that the function M is convex in β as required by Proposition 1(ii). Third, the results in the two propositions hold under any risk preference structure. Risk neutrality and constant absolute risk aversion are special cases, and their corresponding results are contained in the corollary.

Propositions 1 and 2 do not rely on any of the risk preference restrictions that are frequently used in deriving comparative static results in risk models, such as decreasing absolute or increasing relative risk aversion. It is the absence of such preference restrictions that allows the qualitative results in Proposition 2 to be empirically testable. To clarify, consider the proposition: a competitive firm's output supply under price risk is increasing in expected price *if* the firm exhibits nonincreasing absolute risk aversion (NIARA). This result has limited refutable implications. The comparative static result cannot be tested using the parameter estimates from an output supply equation because risk preferences are not directly observable.[3] More importantly, NIARA is sufficient but not necessary for upward sloping supply under price risk. Consequently, if the supply elasticity estimate from an empirical study is found to be negative, one is left in doubt whether the empirical rejection of the comparative static result arises from rejection of the underlying model or from the rejection of NIARA preferences. This problem does not arise in the case of Proposition 2 because quasiconvexity of V is both necessary and sufficient for the testable results contained in the matrix Ω. These results can be verified directly using the parameter estimates of a firm's supply or input demand equations. No additional estimation regarding the firm's risk preference is required.

It may be worth reiterating that, in deriving the results in the two propositions, we have not imposed any distributional restrictions on the random variables either. The only assumptions underlying the two propositions are: (a) structure of wealth in (1) and (2) – which is part of the model specification, and (b) existence of an indirect utility function. The latter is ensured by the existence of a solution to the expected utility maximization problem in (4).

The comparative static and reciprocity results contained in the matrix Ω in Proposition 2 have been derived as refutable implications for the firm models in several studies noted in the preceding section (Chavas and Pope, 1985; Paris, 1988; 1989; Paris *et al.*, 1993). Our contribution lies in demonstrating that the indirect utility function's curvature properties imply and are implied by the refutable behavioural postulates.

III. AN EXPOSITORY EXAMPLE

To shed more light on the analytical results in the preceding section we consider below an n-output firm model under price risk. This generalizes the model in Pope (1980) to the multiple-output case.

Let $\mathbf{Y}$ denote an $n \times 1$ netput vector whose element denotes an output if positive and an input if negative. We assume that the $n \times 1$ random price vector, $\tilde{\mathbf{P}}$, has the following form:

$$\tilde{\mathbf{P}} = \bar{\mathbf{P}} + D \cdot \varepsilon \tag{5}$$

where D is an $n \times n$ diagonal matrix with the parameter vector $\mathbf{d} \equiv \{d_1, \ldots, d_n\}$ on its diagonal, and ε denotes an $n \times 1$ vector of random variables such that $E[\varepsilon_i] = 0$, $\forall i$. The last assumption implies that $E[\tilde{\mathbf{P}}] = \bar{\mathbf{P}}$. Note, if the ith element of the parameter vector $\mathbf{d}$ is zero, then the ith netput price is non-random. Also observe that we have only assumed that each of the random variables in the vector ε has zero mean; no restriction has been imposed on the variance or covariances.

The firm's random wealth is given by:

$$\begin{aligned} \tilde{\mathbf{W}} &= \tilde{\mathbf{P}} \cdot \mathbf{Y} + I \\ &= \bar{\mathbf{P}} \cdot \mathbf{Y} + D \cdot \varepsilon \cdot \mathbf{Y} + I \end{aligned} \tag{6}$$

Thus, in terms of the notation in the preceding section, $M = \bar{\mathbf{P}} \cdot \mathbf{Y}$ in this model, where $\bar{\mathbf{P}}$ corresponds to β and $\mathbf{Y}$ to $\mathbf{x}$.

By Proposition 1(i), the firm's indirect utility function, $V(\bar{\mathbf{P}}, I, \mathbf{d})$, is increasing (decreasing) in $\bar{P}_i$ if the ith element of $\mathbf{Y}$ is an output (input). By Proposition 1(ii), $V(\bar{\mathbf{P}}, I, \mathbf{d})$ is quasiconvex in $\bar{\mathbf{P}}$ and I because M is linear, and therefore, convex in $\bar{\mathbf{P}}$. Further, since $M_{\bar{P}\bar{P}} = 0$, $M_{Y\bar{P}}$ is an identity matrix, and $M_{\bar{P}} = \mathbf{Y}$ in this model, we have the following second-order curvature result by Proposition 2:

$$\begin{aligned} &V(\bar{\mathbf{P}}, I, \mathbf{d}) \text{ quasiconvex in } \bar{\mathbf{P}} \text{ and} \\ &I \Leftrightarrow \Omega \equiv \mathbf{Y}^*_{\bar{P}} - \mathbf{Y}^*_I \cdot \mathbf{Y}^* \text{ is SPSD,} \end{aligned} \tag{7}$$

where $\mathbf{Y}^*$ denotes the optimal netput vector. Thus, the signs of the indirect utility function's first derivatives, and the

[3]In this context, Silberberg's comment is noteworthy: 'If the theory is to be at all useful, the assumptions, or test conditions, must be *observable*' (p. 10, emphasis in original).

symmetry and positive semidefiniteness of the matrix $\Omega \equiv \mathbf{Y}^*_{\bar{P}} - \mathbf{Y}^*_I \cdot \mathbf{Y}^*$ contains the complete set of refutable implications for the multiple-output firm model under price risk.

To see the output supply and input demand restrictions implied by the foregoing results, partition the netput vector into its output and input components: $\mathbf{Y} = \{\mathbf{Q}, -\mathbf{x}\}$ with the corresponding expected price vector being conformably partitioned: $\bar{\mathbf{P}} \equiv \{\mathbf{p}, \mathbf{r}\}$. Thus, the Proposition 1(i) result translates to:

$$V_{\mathbf{p}} \overset{S}{=} \mathbf{Q}^* > 0 \tag{8a}$$

$$V_{\mathbf{r}} \overset{S}{=} -\mathbf{x}^* < 0 \tag{8b}$$

where $\overset{S}{=}$ denotes 'same sign as'. The result in (8), implied by the first-order curvature properties of the indirect utility function, can be viewed as the counterpart to monotonicity of the profit function under certainty. Also, by the result in (7):

$$\Omega \equiv \mathbf{Y}^*_{\bar{P}} - \mathbf{Y}^*_I \cdot \mathbf{Y}^* \equiv \begin{bmatrix} Q^*_p & Q^*_r \\ -\mathbf{x}^*_p & -\mathbf{x}^*_r \end{bmatrix} - \begin{bmatrix} \mathbf{Q}^*_I \cdot \mathbf{Q}^* & \mathbf{Q}^*_I \cdot \mathbf{x}^* \\ -\mathbf{x}^*_I \cdot \mathbf{Q}^* & \mathbf{x}^*_I \cdot \mathbf{x}^* \end{bmatrix} \text{ is SPSD} \tag{9}$$

By positive semidefiniteness of Ω:

$$\mathbf{Q}^*_p - \mathbf{Q}^*_I \cdot \mathbf{Q}^* \text{ is SPSD} \tag{10a}$$

$$\mathbf{x}^*_r + \mathbf{x}^*_I \cdot \mathbf{x}^* \text{ is symmetric negative semidefinite (SNSD)} \tag{10b}$$

And, by symmetry of Ω, we have the following reciprocity result:

$$\mathbf{Q}^*_r - \mathbf{Q}^*_I \cdot \mathbf{x}^* \equiv -\mathbf{x}^*_p + \mathbf{x}^*_I \cdot \mathbf{Q}^* \tag{10c}$$

The results in (8) and (10) are for the multiple-output firm. The corresponding results for a single-output firm are special cases of these. For example, when Q is a scalar, the result in (10a) becomes:

$$Q^*_p - Q^*_I \cdot Q^* \geqslant 0 \tag{10a$'$}$$

It will be seen in the following section that the qualitative results contained in (8) and (10) can be easily translated into empirically testable parameter restrictions on a firm's demand and supply equations.

It may be worth noting that the model set out above requires only minor changes to accommodate output risk instead of price risk. For example, if one assumes that the firm's production function is of the Just-Pope type (see footnote 2), then the wealth function becomes:

$$\tilde{\mathbf{W}} = pf(x) + pD \cdot \varepsilon \cdot g(\mathbf{x}) - \mathbf{r} \cdot \mathbf{x} + I \tag{6$'$}$$

where a nonzero ith element of the parameter vector $\mathbf{d}$ now implies that the ith output is produced through a random production process, and p denotes the output price. As before, ε is assumed to have zero mean. Consequently, the function M is given by $pf(\mathbf{x}) - \mathbf{r} \cdot \mathbf{x}$, and β is the vector of input prices, $\mathbf{r}$. This implies that the results in (8b) and (10b) hold under output risk, and under price *and* output risk.

IV. AN EMPIRICAL APPLICATION

The complete set of refutable implications for a competitive firm under price risk is tested using data for Kansas wheat farmers. The data consist of 107 firm-level annual observations of quantities and prices for output and two aggregate inputs (capital and materials) for the period, 1973–90. The primary data were from the computerized farm accounting records of the Farm Management Data Bank, Department of Agricultural Economics, Kansas State University (Langemeier, 1990). Each farm in the sample devoted at least 95% of row crop acreage to wheat and received at least 90% of farm income from the sale of wheat. Thus, these farms can be effectively regarded as single-output producers.

Data included expenditures for a nearly exhaustive array of inputs which were aggregated into two input categories, capital and materials. The data also included government payments and nonfarm income. Because some input prices were not available from the Data Bank farm-level data, they were supplemented with Kansas state-level price data (US Department of Agriculture). Data on wheat price were generated by dividing the sum of revenue from wheat sales and government deficiency payments by wheat output for each farm.

We used single period lagged wheat price as the estimate for producers' expected output price. This procedure warrants some explanation. Following the quasi-rational expectations approach (Nerlove *et al.*, 1979), many empirical studies have used ARIMA analysis on time series data to generate price moments. Others have utilized the alternative framework of adaptive expectations (Just, 1974). However, in our empirical analysis, which uses cross-sectional data, adoption of either of these procedures would impose the restriction that all farmers face the same output price and, therefore, have the same price expectations. Largely as a result of the government deficiency payments, the output price received varied considerably by farms within the cross-section. Therefore, we concluded that it would be inappropriate to use state-level time series data on Kansas wheat prices to generate the expected price data. We assumed, instead, that the random output price received by Kansas producers is characterized by a Markov process, which implies that all relevant information about the next period's expected price is contained in the current period's realization. The usage of single period lagged wheat price for expected price is consistent with several empirical studies (see, for example, Chavas and Holt, 1990, and the references therein).

Expenditures on capital inputs, denoted by x_1, included interest charges on land and building equity, cash farm rent, building and machinery depreciation, and real estate taxes. The materials category, denoted by x_2, included machinery and machinery hire, fertilizer, pesticides, seed, and miscellaneous cash expenses. Because of incomplete data, labour was not included in the list of inputs.[4] All price aggregates were computed as geometric means using expenditure or revenue shares as the weights. A copy of the data used and additional details about data construction are available on request from the authors.[5]

The following equations were estimated as a system of three seemingly unrelated equations:

$$Q = a + \mathbf{xB} + 0.5\mathbf{xCx} + \mathbf{e}_0 \qquad (11a)$$

$$x_j = \alpha_j + \mathbf{Z}\phi_j + 0.5\mathbf{Z}\Gamma_j\mathbf{Z} + \mathbf{e}_j \qquad j = 1, 2 \qquad (11b)$$

where Q is output quantity; x_j is the jth input, $\mathbf{x}$ is the vector $\{x_1, x_2\}$; and the vector $\mathbf{Z} = \{p, r_1, r_2, I\}$ contains data on expected output price, the two input prices, and off-farm income. The vector or matrices of parameters to be estimated are: a, $\mathbf{B}$, C, α_j, ϕ_j, and Γ_j. The error terms are denoted by e_0 and e_j. The first equation in the system is the production function while the remaining two are the input demand equations. Because the production technology and the form of the input demand equations are unknown, the quadratic form – a second-order Taylor-series approximation to the underlying 'true' form – was used. No restrictions were imposed on the estimated parameters so that all refutable implications under price uncertainty contained in (8a), (8b), (10a′), (10b), and (10c) could be tested. All hypotheses were tested locally at the sample means as well as at each observation.

The transformation of the analytical results to parameter restrictions warrants some explanation. Consider, for example, the expression for the derivative Q_p in terms of the parameters of the system in (11). Since $Q_p = Q_x x_p^*$, we begin by differentiating (11a) with respect to x_j:

$$Q_{x_j} = \mathbf{B}_j + C_j\mathbf{x} \qquad j = 1, 2 \qquad (12a)$$

where $\mathbf{B}_j$ and C_j are the jth row of the parameter vector $\mathbf{B}$ and the matrix C. Differentiation of the jth equation in (11b) with respect to p yields:

$$x_{j_p} = \phi_j^p + \Gamma_j^p\mathbf{Z} \qquad j = 1, 2 \qquad (12b)$$

where ϕ_j^p and Γ_j^p denote the row corresponding to p of the vector ϕ_j and the matrix Γ_j in the jth equation. Combining (12a) and (12b), the expression for Q_p in terms of estimated parameters of the system in (11) becomes:

$$\hat{Q}_p = \sum_{j=1}^{2} (\hat{\mathbf{B}}_j + \hat{C}_j\mathbf{x}) \cdot (\hat{\phi}_j^p + \hat{\Gamma}_j^p\mathbf{Z}) \qquad (13)$$

where the data vectors $\mathbf{x}$ and $\mathbf{Z}$ are evaluated at sample means. Steps similar to the ones outlined above were used in all transformations of analytical expressions in (8a), (8b), (10a′), (10b), and (10c) to parameter restrictions.

The R^2 for the three equations in (11) were 0.86, 0.28 and 0.26, respectively. The likelihood ratio test for a diagonal covariance matrix for the equation system was 168.23 (d.f. = 3), clearly rejecting the null. In the interest of brevity, we are not reporting the 36 parameter estimates from (11), but they are available from the authors on request. Only the test results of the refutable implications are presented in Table 1.

The first three rows of Table 1 contain results on the signs of the first derivatives of the indirect utility function [(8a) and (8b)]. It is evident that V is indeed increasing in expected output price and decreasing in the two input prices. At the data means, predicted output and input levels were significantly positive. In addition, predicted quantities were significantly positive at the 5% level of significance for 106 observations of output level, for 100 observations of x_1 and for 99 observations of x_2. None of the predicted quantities was significantly negative for any observation.

Rows 2a–2e contain test results of refutable implications implied by the second-order curvature properties of V. None of the behavioural postulates implied by positive semidefiniteness of Ω is refuted at the means at the 5% level significance. Although the sign of the comparative static expression in 2b is positive at the means, it is not significantly different from zero. Except for expression 2b, there was little evidence of curvature violations of Ω at individual observations. Even in the case of 2b, the compensated, own-price, input demand responses were significantly positive in less than 25% of the observations.

The test result implied by symmetry of the matrix Ω is contained in row 3. The three symmetry restrictions are not rejected by the joint test at the means or at 103 of the 107 observations. In sum, none of the refutable implications implied by the first and second-derivative curvature

[4]The accounting data did not include quantity of family labour. A few farms computed a charge for unpaid family labour, but most did not. All farms reported the number of hired workers but only in integers, some of which were suspect. There were no data reported on hours, weeks, or months worked by hired employees. Therefore, the labour data were judged to be too unreliable or inadequate to include in the analysis.

[5]Appreciation is extended to Larry Langemeier and the Kansas State University faculty for access to the extensive data in the Farm Management Data Bank. Any release of data used in this study is subject to the conditions for research data access maintained by the Kansas Farm Management Associations. These conditions include anonymity of individual observations. Observations are identified in our data set only by observation number and not by farm or regional code.

Table 1. *Test results of the refutable implications of the firm model under price risk*

Restrictions/Null hypotheses	Test type	Test at data means Statistic	*p*-value	Rejections among 107 observations[b]
1. First derivative conditions				
1a. V is increasing in p $\hat{Q} > 0$	AN	22.255	0.000	0
1b. V is decreasing in r_1 $\hat{x}_1 > 0$	AN	17.563	0.000	0
1c. V is decreasing in r_2 $\hat{x}_2 > 0$	AN	47.167	0.000	0
2. Positive semi-definiteness of Ω:				
2a. $Q_P^* - Q_I^* \cdot Q^* \geqslant 0$	AN	1.057	0.145	3
2b. $x_{1_{r_1}}^* + x_{1_I}^* \cdot x_1^* \leqslant 0$	AN	1.018	0.154	24
2c. $x_{2_{r_2}}^* + x_{2_I}^* \cdot x_2^* \leqslant 0$	AN	−3.265	1.000	0
2d. Principal minor of $\Omega \geqslant 0$	AN	1.121	0.131	1
2e. Determinant of $\Omega \geqslant 0$	AN	0.309	0.488	0
3. Symmetry of Ω[a]:	WC	7.744	0.052	4
4. CARA or RN $x_{1_I}^* = x_{2_I}^* = 0$	WC	6.853	0.033	28

Codes: AN = Asymptotic normal
WC = Wald chi-squared
CARA = Constant absolute risk aversion
RN = Risk neutrality

[a]The test of symmetry involves the joint test:

$$H_0: x_{1_{r_2}}^* + x_{1_I}^* \cdot x_2^* = x_{2_{r_1}}^* + x_{2_I}^* \cdot x_1^*$$
$$\text{and } Q_{r_1}^* - Q_I^* \cdot x_1^* = -x_{1_P}^* + x_{1_I}^* \cdot Q^*$$
$$\text{and } Q_{r_2}^* - Q_I^* \cdot x_2^* = -x_{2_P}^* + x_{2_I}^* \cdot Q^*$$

[b]At 0.05 level of significance.

properties of the indirect utility function are rejected at the data means for these Kansas wheat producers, but one of the implications is rejected for nearly 25% of the individual observations.

The last set of test results show that the hypothesis of either risk neutrality or constant absolute risk aversion is clearly rejected at the data means. However, the rejection is less pronounced at individual observations: it is rejected in just over 26% of the total observations.

V. CONCLUDING COMMENTS

The objectives of this paper were to derive the complete set of indirect utility function properties for a competitive firm under price or output risk and to empirically test the behavioural postulates implied by these properties. The paper's main analytical contribution lies in demonstrating that the indirect utility function's first and second-order curvature properties are both necessary and sufficient for refutable behavioural postulates in the form of comparative static results, reciprocity relations, and restrictions on output and input responses. Importantly, these postulates are independent of restrictions on risk preference, technology, or distribution of random variables within the wealth function. All that is needed is the existence of an optimal solution to the firm's expected utility maximization problem. As an exposition, the refutable implications for a multiple-output firm operating under price risk were derived. It was also demonstrated that minor changes in the model can accommodate output risk instead of price risk. In the paper's empirical section, an estimation framework for testing the complete set of refutable implications for a firm model under price risk was proposed and applied to firm-level data. The data analysis did not reject any of the refutable restrictions on output supply and input demand responses, comparative static results, and symmetry restrictions at the data means. Risk neutrality and constant absolute risk aversion were both rejected at the means. At individual observations, however, risk neutrality and constant absolute risk aversion were rejected only slightly more frequently than was one of the refutable implications of expected utility maximization.

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APPENDIX

Proof of Proposition 1

(i) Differentiating V in (4) and applying the envelope theorem yields:

$$V_\beta = E[u'] \cdot M_\beta$$

$$V_I = E[u']$$

which completes the proof since $E[u'] > 0$. □

The following result must precede the proof of Proposition 1(ii).

Claim 1. The problem $\mathrm{Max}_x\{E[u(M(x, \beta, \cdot) + S(x; \tilde{\varepsilon}, \cdot) + I)]\}$ can be equivalently expressed as a constrained optimization problem where x and $\bar{W}$ are jointly chosen. In particular, define $\mathbf{z} = \{\mathbf{x}, \bar{W}\}$, $\theta \equiv \{\beta, I\}$; then:

$$\begin{aligned} V(\beta, I, \cdot) &= \underset{x}{\mathrm{Max}}\,\{E[u(M(x; \beta, \cdot) + S(x; \tilde{\varepsilon}, \cdot) + I)]\} \\ &= \underset{z}{\mathrm{Max}}\,\{E[u(\bar{W} + S(x; \tilde{\varepsilon}, \cdot))] \mid \bar{W} \\ &\leqslant M(x; \beta, \cdot) + I\} \end{aligned} \tag{A1}$$

Proof. To prove claim 1 we first demonstrate that the constraint in (A1) will be binding for all optimal values of $\bar{W}$ and x. We prove this by contradiction. Suppose the constraint is not binding. That is, suppose at some arbitrary parameter values $\theta^\circ \equiv \{\beta^\circ, I^\circ\}, z^\circ \equiv \{x^\circ, \bar{W}^\circ\}$ is optimal but $\bar{W}^\circ < M(x^\circ, \beta^\circ, \cdot) + I^\circ$, i.e. the constraint is nonbinding. Then there exists some $\bar{W}' > \bar{W}^\circ$ such that $\bar{W}' = M(x^\circ, \beta^\circ, \cdot) + I^\circ$; therefore

$\{\bar{W}', x^\circ\}$ is feasible. Since u is increasing by assumption, we have $E[u(\bar{W}' + S(x^\circ; \tilde{\varepsilon}, \cdot))] > E[u(\bar{W}^\circ + S(x^\circ; \tilde{\varepsilon}, \cdot))]$. But then z° could not have been optimal. The proof of Claim 1 now follows from the substitution of the binding constraint $\bar{W} = M(x; \beta) + I$ into the objective function of the constrained optimization problem in (A1) and by noting that θ° was any arbitrary parameter value. □

We now proceed to prove Proposition 1(ii). First define:

$$H(z; \theta) \equiv \bar{W} - M(x; \beta; \cdot) - I \leqslant 0 \qquad \text{(A2)}$$

as the constraint in the problem:

$$V(\theta, \cdot) \equiv \operatorname*{Max}_{z} \{E[u(\bar{W} + S(x; \tilde{\varepsilon}, \cdot))] \,|\, H(z; \theta) \leqslant 0\} \qquad \text{(A1')}$$

If $M(x; \beta, \cdot)$ is convex in β, $H(z; \theta)$ is concave and hence quasiconcave in θ. Let θ', θ'' and $\bar{\theta}$ be any feasible parameter values such that: $\bar{\theta} = t\theta' + (1-t)\theta''$, $0 \leqslant t \leqslant 1$. Let $\bar{z}$ denote the optimal choice vector corresponding to $\bar{\theta}$. Since $H(z; \theta)$ is quasiconcave in θ, and $\bar{\theta}$ and $\bar{z}$ satisfy the constraint in (A2), it follows from the definition of quasiconcavity that:

$$0 \geqslant H(\bar{z}, \bar{\theta}) \geqslant \min\{H(\bar{z}, \theta'), H(\bar{z}, \theta'')\} \qquad \text{(A3)}$$

The inequality in (A3) implies either $H(\bar{z}, \theta') \leqslant 0$ or $H(\bar{z}, \theta'') \leqslant 0$ or both. Therefore, $\bar{z}$ is feasible when either $\theta = \theta'$ or $\theta = \theta''$, or both, but not optimal, since $\bar{z}$ is optimal only when $\theta = \bar{\theta}$. Therefore,

$$V(\bar{\theta}, \cdot) \leqslant \max\{V(\theta', \cdot), V(\theta'', \cdot)\} \qquad \text{(A4)}$$

But the inequality in (A4) implies, by definition, that $V(\cdot)$ is quasiconvex in $\theta \equiv \{\beta, I\}$. □

Proof of Proposition 2

Application of the envelope theorem to the problem in (A1) yields:

$$M_\beta(x^*(\beta, I, \cdot); \beta; \cdot) \equiv V_\beta / V_I \qquad \text{(A5)}$$

Differentiate both sides of (A5) first with respect to β and then with respect to I:

$$M_{\beta\beta} + M_{x\beta} \cdot x^*_\beta \equiv (V_{\beta\beta} \cdot V_I - V_\beta V_{I\beta}) / V_I^2 \qquad \text{(A6a)}$$

$$M_{x\beta} \cdot x^*_I \equiv (V_{\beta I} \cdot V_I - V_\beta V_{II}) / V_1^2 \qquad \text{(A6b)}$$

Multiply both sides of (A6b) by $M_\beta \equiv V_\beta / V_I$ and subtract the resulting expression from (A6a):

$$M_{\beta\beta} + M_{x\beta} \cdot (x^*_\beta - x^*_I \cdot M_\beta) \equiv \frac{1}{(V_I)^3}\{V_{\beta\beta} V_1^2 - 2V_\beta V_{I\beta} V_I + V_{II} V_\beta V_\beta\} \qquad \text{(A7)}$$

By definition of quasiconvexity of V in β and I, and by Young's Theorem the right-hand side of (A7) constitutes a symmetric, positive semidefinite (SPSD) matrix. □

Proof of Corollary

Consider again the optimization problem in (A1):

$$\operatorname*{Max}_{x} H \equiv E[u(M(x; \beta, \cdot) + S(x; \tilde{\varepsilon}; \cdot) + I)] \qquad \text{(A8)}$$

Let $D \equiv H_{xx}(x^*)$ denote the hessian matrix. Assume that the second-order sufficient condition for the problem is satisfied and D is negative definite. Then:

$$x^*_I \equiv H_{xI} \cdot (-D)^{-1} = E[u'' \cdot \{M_x + S_x\}] \cdot (-D)^{-1} \qquad \text{(A9)}$$

Now observe that $u'' = -u' \cdot \bar{A}$, where $\bar{A}$ denotes the Arrow–Pratt measure of risk aversion that is a constant under constant absolute risk aversion (CARA), and is equal to zero under risk neutrality (RN). Clearly, therefore, under RN, $x^*_I \equiv 0$. For the result under CARA, substitute $u'' = -u' \cdot \bar{A}$ into (A9):

$$x^*_I = -\bar{A} E[u' \cdot \{M_x + S_x\}] \cdot (-D)^{-1} = 0 \qquad \text{(A10)}$$

where the last equality follows from the first order condition of (A8). Now recall that quasiconvexity of $M_{\beta\beta} + M_{x\beta}(x^*_\beta - x^*_I M_\beta)$ in (A7) holds for all preference structures, including RN and CARA. Substitution of (A10) into (A7) completes the proof. □