PRODUCTION AND SAVINGS UNDER UNCERTAINTY

ATANU SAHA, ROBERT INNES, RULON POPE

ABSTRACT

This paper studies an integrated model of production and savings under uncertainty in which production inputs and the amount of savings are jointly chosen. The analysis shows that if the agent's risk preferences exhibit constant absolute risk aversion, then all results from nonintegrated or separate models of savings and production extend to the integrated framework. Under decreasing absolute risk aversion, the comparative static properties of optimal production decisions with respect to mean preserving spread and spread preserving mean parameters extend from the non-integrated to the integrated framework. However, extension of the savings model results for the same parameters requires a restriction on production technology.

I. INTRODUCTION

In the early 1970s the literature on uncertainty saw two path breaking contributions by Sandmo. The first analyzed the savings choices of an agent under uncertainty (Sandmo,

Direct all correspondence to: Atanu Saha, Assistant Professor of Agricultural Economics, Texas A&M University, College Station, TX 77843-2124; Robert Innes, Professor of Agricultural Economics, University of Arizona, Tucson, AZ 85721; Rulon Pope, Professor of Economics, Brigham Young University, Provo, UT 84602.

International Review of Economics and Finance, 2(4) 365–375 Copyright © 1993 by JAI Press, Inc.
ISSN: 1059-0560 All rights of reproduction in any form reserved.

1970), and the other studied the decision problems of a competitive firm under price uncertainty (Sandmo, 1971). Each of these papers inaugurated extensive—but distinct—literatures on production and savings choice problems.[1] This paper studies an integrated model of production and savings under uncertainty to explore the linkages between these two decision problems. In particular, it analyzes the comparative static properties of optimal input and savings choices and derives the conditions under which standard results from separate models of production and savings under uncertainty extend to the integrated framework.

The applications of this paper's model framework are numerous. Consider, for example, the standard model of production under output or price uncertainty: A risk averse producer maximizes her expected utility function, $E[U(\tilde{y})]$, by choosing the optimal levels of inputs or output. In the uncertainty literature, this model is typically set in a static and timeless environment for the sake of analytical simplicity. However, the actual choice environment of the producer is not timeless. In fact, one is hard-pressed to find even one example of a risk-averse producer who does not make the joint decisions of production *and* intertemporal income transfer through savings or investment, whether it be a business entrepreneur, a farmer, or a professional service provider. Therefore, in constructing a theory of decision-making for such agents, it is important to consider the implications of integrating savings and production choice problems.

The analysis shows that if an agent exhibits constant absolute risk aversion (CARA), then *all* comparative static results from standard, non-integrated models of savings and production extend to the integrated model. Under the more plausible assumption of decreasing absolute risk aversion (DARA), the comparative static properties of optimal production decisions with respect to mean preserving spread and spread preserving mean parameters extend from the non-integrated to the integrated framework. However, extension of the savings model results for the same parameters requires a restriction on production technology.

II. THE NON-INTEGRATED AND INTEGRATED MODELS OF PRODUCTION AND SAVINGS

This section presents three models: the first two, Models A and B, are the standard, separate models of production and savings, while the third, Model C, is the integrated model wherein production and savings decisions are made jointly. This section demonstrates that any comparative static result carries over from Models A and B to C when risk preferences exhibit CARA.

A. *Model A: Production Under Multiplicative Output Risk Without Savings*[2]

It is assumed that an agent maximizes the expected utility of random income, $\tilde{y}$, which is the sum of random profit and risk-free exogenous income, N:

$$\underset{x}{Max}\, A \equiv E[U(\tilde{y})] \equiv E[U(pF(X, \tilde{\varepsilon}) - w^T X + N)] \tag{1}$$

where E denotes the expectation operator, $U(\cdot)$ is a Von-Neumann-Morgenstern utility function, assumed to be strictly concave in $\tilde{y}$, and p denotes output price. $F(\cdot)$: $\mathbf{R}^n \times \mathbf{R} \rightarrow \mathbf{R}$, is the stochastic production function, where $X \in \mathbf{R}^n_+$ is the vector of inputs whose prices are denoted by $w \in \mathbf{R}^n$ and $\tilde{\varepsilon}$ is a random variable with support on $\mathbf{R}_+$. It is assumed that $A(\cdot)$ is twice differentiable, with interior solutions satisfying:

$$A_x \equiv E[U' \{pF_x(X,\tilde{\varepsilon}) - w\}] = \underline{0}\,, \tag{2}$$

and that the second order sufficient condition is satisfied, i.e., A_{xx^T}, evaluated at the optimum, is negative definite. Denote the solutions to equation (2) as $X^*(a)$, where $a \equiv (p,w,N,\alpha)$ is the parameter vector, α being the moments of the distribution of $\tilde{\varepsilon}$.

B. Model B: Savings Under Income Uncertainty Without Production

The agent maximizes a two-period additively separable utility function where income is transferred from the first period to the second through savings, S:

$$\underset{S}{Max}\, B = U_1(I_1 - S) + E[U_2(S(1 + r) + \tilde{M}) \tag{3}$$

where I_1 denotes first period wealth, r the interest rate and $\tilde{M}$ is the random part of the second period income. It is assumed that $B(\cdot)$ is twice differentiable, with interior solutions satisfying:

$$B_s = -U_1' + E[U_2'\{1 + r\}] = 0, \tag{4}$$

and that $B_{ss}(S^*(b)) < 0$, where $S^*(b)$ denotes the optimal level of savings, $b \equiv (I_1,r,\beta)$ is the parameter vector, and β the moments of $\tilde{M}$.

C. Model C: The Integrated Model of Production and Savings

In this model, the optimal production inputs and the amount of income to be transferred from the first to the second period in the form of savings are jointly chosen. These decisions are made before production uncertainty is resolved.

$$\underset{X,S}{Max}\, C \equiv U_1(I_1 - S) + E[U_2(pF(X, \tilde{\varepsilon}) - w^T X + (1 + r)S)]\,. \tag{5}$$

All variable and parameters in equation (5) have the same definition as in Models A and B. Clearly, Model C nests Models A and B as special cases. If S is held fixed in equation (5), with X being the only choice variable, then the resulting restricted model is mathematically equivalent to Model B, with N set equal to $(1 + r)S$. Similarly, if X is held fixed in equation (5) then the resulting model is equivalent to Model C, with $\tilde{M}$ set equal to $(pF(X)\tilde{\varepsilon} - w^T X)$. Denote the solutions of equation (5) by $S^*(c)$ and $X^*(c)$ where $c = (a,I_1,r)$ is the parameter vector. To relate Models A and B to the integrated Model C, we now turn to the characterization of Model C's solutions, beginning with the first and second order sufficient conditions (again assuming interior solutions):

$$C_x\,(S^*(c), X^*(c)\,) = E[U_2'\{pF_x(X, \tilde{\varepsilon}) - w\}] = \underline{0} = A_x \tag{6a}$$

$$C_s\,(S^*(c), X^*(c)\,) = -U_1' + E[U_2'\{1 + r\}] = 0 = B_s \tag{6b}$$

$$H = \begin{pmatrix} C_{xx^T} & C_{xs} \\ C_{sx^T} & C_{ss} \end{pmatrix}, \text{ evaluated at } (S^*(c), X^*(c)), \text{ is negative definite,} \tag{7}$$

where:

$$C_{xx^T} = E[U_2''\{pF_x(X,\tilde{\varepsilon}) - w\}^{\mathrm{T}}\{pF_x(X,\tilde{\varepsilon}) - w\}] + E[U_2'\,pF_{xx^T}(X,\tilde{\varepsilon})] = A_{xx^T} \tag{7a}$$

$$C_{ss} = U_1'' + E[U_2''\{1 + r\}^2] = B_{ss} < 0 \tag{7b}$$

$$C_{xs} = = E[U_2''\{pF_x(X,\tilde{\varepsilon}) - w\}(1 + r)] \tag{7c}$$

Substitution of $S^*(c)$ and $X^*(c)$ into equation (6) and differentiating the resulting identities with respect to c_i, the i-th element of c, yields the following comparative static expression:

$$\begin{pmatrix} X^*_{c_i} \\ S^*_{c_i} \end{pmatrix} = (-H^{-1}) \begin{pmatrix} C_{xc_i} \\ C_{sc_i} \end{pmatrix} \tag{8}$$

We now proceed to characterize the conditions under which the comparative static properties of Models A and B extend to C.

Lemma 1: (a) If U_2 exhibits CARA everywhere, then $C_{xs} = \underline{0}$. (b) If U_2 exhibits decreasing absolute risk aversion DARA, then $C_{xs} > \underline{0}$.

The proof of this and other Lemmas and Propositions that follow are presented in the Appendix.

Proposition 1: Suppose U_2 ( ) exhibits CARA everywhere, then:

(i) $X^*_{c_i}$ is $> \underline{0}$ ($\underline{0}$, $< \underline{0}$) in Model C if and only if $X^*_{a_i}$ is $> \underline{0}$ ($\underline{0}$, $< \underline{0}$) in Model A with $N \equiv (1 + r)S^*(b)$.

(ii) $S^*_{c_i}$ is > 0 (0, < 0) in Model C if and only if $S^*_{b_i}$ is > 0 (0, < 0) in Model B with $\tilde{M} \equiv pF(X^*(a),\tilde{\varepsilon}) - w^T X^*(a)$.

It is clear from Proposition 1 that the structure of risk preferences is pivotal in the problem of extending qualitative results from the non-integrated to the integrated framework. The intuition for this is as follows. In the integrated model, optimal choices are affected not only directly by any parameter change but also indirectly through the change in second period income induced by savings or production responses to the parameter change. Whether and how this change in income in turn influences the optimal choices depends on the relationship between the agent's risk preference structure and her income. Under CARA, since the agent's aversion to risk is independent of income level, the change in second period income has no effect on optimal choices. Consequently, the "indirect" effect of the parameter change is absent. Under DARA preferences, however, any income change affects the agent's attitude toward risk and, therefore, also her optimal choices. The resulting interaction between the "direct" and "indirect" responses under DARA prevents, *in general*, the direct extension of comparative static results from the non-integrated to the integrated model framework. However, as will be seen in the following section, in particular cases this extension under DARA is possible without additional restrictions.

III. COMPARATIVE STATICS UNDER DECREASING ABSOLUTE RISK AVERSION

Under DARA preferences, there are four well-known comparative static results from Models A and C that are set out below. In this section, we investigate whether and when these comparative static results hold in the integrated Model C when the agent exhibits DARA preferences. Our attention will be restricted to the mean preserving spread (MPS) and spread preserving mean (SPM) parameters. We do not investigate the effects of prices, p and r, primarily because even in extant models of production and savings these parameters elicit ambiguous responses (e.g., see Pope and Kramer, 1979; Sandmo, 1970).

We proceed by first defining the MPS and SPM parameters in Models A and B. In the production model we assume that the stochastic production function has the following form:

$$F(X,\tilde{\varepsilon}) = F(X)\tilde{\varepsilon} \tag{9a}$$

where $\tilde{\varepsilon} = \phi + \gamma\tilde{e}$; and in the savings model, the stochastic income is given by:

$$\tilde{M} = \phi + \gamma\tilde{e}. \tag{9b}$$

In both cases it is assumed that $E[\tilde{e}] = 0$, implying ϕ is the SPM and γ the MPS parameter. Also, for the sake of simplicity, we assume in this section that X is a scalar. The following comparative static properties of Models A and B now provide our point of departure (see Sandmo, 1970 and Batra and Ullah, 1974 for discussion and proofs):

Model A:

$$X_\phi^* \stackrel{S}{=} pF'E[U_2'] + pFE[U_2''z] > 0, \qquad \text{Result (1)}$$

$$X_\gamma^* \stackrel{S}{=} pF'E[U_2'\tilde{e}] + pFE[U_2''z\tilde{e}] < 0. \qquad \text{Result (2)}$$

Model B:

$$S_\phi^* \stackrel{S}{=} (1+r)E[U_2''] < 0 \qquad \text{Result (3)}$$

$$S_\gamma^* \stackrel{S}{=} (1+r)E[U_2''\tilde{e}] > 0 \qquad \text{Result (4)}$$

where $z \equiv \{pF'\tilde{\varepsilon} - w\}$, and $\stackrel{S}{=}$ denotes "same sign as". We now proceed to investigate the conditions under which results (1) to (4) extend to the integrated model.

Model C:

It follows from equations (7) and (8) that:

$$X_\phi^* \overset{S}{=} -C_{ss}C_{x\phi} + C_{xs}C_{s\phi} \tag{10a}$$

$$S_\phi^* \overset{S}{=} -C_{xx}C_{s\phi} + C_{xs}C_{x\phi} \tag{10b}$$

where:

$$C_{x\phi} = pF'E[U_2'] + pFE[U_2''z] > 0 \tag{11a}$$

$$C_{s\phi} = (1 + r)pFE[U_2''] < 0\,. \tag{11b}$$

The inequality in equation (11a) follows from Lemma 1. Note, under CARA preferences, $C_{xs} = 0$; hence, $X_\phi^* > 0$ and $S_\phi^* < 0$, as implied by Proposition 1 and results 1 and 3 above.

Similarly from equations (7) and (8):

$$X_\gamma^* \overset{S}{=} -\; C_{ss}C_{x\gamma} + C_{s\gamma}C_{xs} \tag{12a}$$

$$S_\gamma^* \overset{S}{=} -\; C_{s\gamma}C_{xx} + C_{xs}C_{x\gamma} \tag{12b}$$

where:

$$C_{x\gamma} = pF'E[U_2'\tilde{e}] + pFE[U_2''z\tilde{e}] \tag{13a}$$

$$C_{s\gamma} = (1+ r)pFE[U_2''\,\tilde{e}]\,. \tag{13b}$$

The terms in equation (13) are signed by:

Lemma 2: (i) $C_{s\gamma} > 0$, and (ii) $C_{x\gamma} < 0$.

Under CARA preferences since $C_{xs} = 0$, $X_\gamma^* < 0$ and $S_\gamma^* > 0$, as implied by Proposition 1 and results 2 and 4 above.

For the case of DARA preferences, inspection of equations (10) and (12) reveals that the "direct" and the "indirect" effects of each parameter change are of opposite signs in the case of both, savings and production results. For example, in equation (12a), the "direct-effect" component of X_γ^* is captured by the term, $-\,C_{ss}C_{x\gamma}$, which is negative by Lemma 2 and second order conditions; the "indirect-effect" is captured by $C_{xs}C_{s\gamma}$, which is positive from Lemma 2 and DARA preferences ($C_{xs} > 0$). However, simplification of the comparative static expressions in equations (10a) and (12a) reveals that results (1) and (2) from the non-integrated production model extend unconditionally to the integrated model under DARA. However, the savings model results, equations (3) and (4), extend to the integrated framework only under a restriction on the production function. These findings are summarized in the following proposition:

Proposition 2: In Model A, under DARA:

(a) $X_\phi^* > 0$ and $X_\gamma^* < 0$,

(b)[3] $S_\phi^* < 0$ and $S_\gamma^* > 0$, if at the optimum, $(FF'' + F'^2) < 0$.

The condition in Proposition 2b is the sufficient condition for the extension of the savings model results. Weaker necessary conditions are presented in the proof of Proposition 2 (Appendix). The condition in Proposition 2b can be stated alternatively as: $\{\frac{F''}{F'}X + \frac{F'}{F}X\} < 0$, which implies $\varepsilon(F') + \varepsilon(F) < 0$, where ε denotes elasticity. For the Cobb-Douglas production function, $F(X) = X^\mu$, this condition does not hold globally; it is satisfied only if $\varepsilon(F) = \mu < 1/2$. An example of a strictly concave production function that *does* satisfy the condition globally is:

$$F(X) = \left\{\gamma - e^{(\delta - X)}\right\}^{1/2} \tag{14}$$

It is readily verified that, for this production function, $(FF'' + F'^2) = \frac{-e^{(\delta - X)}}{2} < 0$.

To gain some intuition into why production results extend to the integrated framework unconditionally while the savings results do not, observe that, in the integrated model, changes in savings and optimal input have qualitatively different effects on the second period income, $\tilde{y}_2$. In particular, in response to a parameter change, the change in S^* and the attendant "indirect" effect on X^*, is only a mean effect. It changes the expected value of $\tilde{y}_2$ leaving its 'riskiness' or variance unchanged. In contrast, a X^* response changes the mean *and* the variance of $\tilde{y}_2$; therefore the "indirect" effect on S^* has a conflicting risk-effect, in addition to the mean effect. The sufficient condition in Proposition 2 offsets the conflicting part of the "indirect effect" to yield an unambiguous optimal savings response.

IV. CONCLUDING REMARKS

The principal finding of this study is that the comparative static results from separate or "non-integrated" models of production and savings under uncertainty do not necessarily extend to an integrated framework wherein agents undertake production and savings decisions simultaneously. A condition which is *sufficient* for this extension is that the producer's risk preferences exhibit constant absolute risk aversion (CARA). Under the more plausible assumption of decreasing absolute risk aversion (DARA), the comparative static exercises with respect to the spread preserving mean and mean preserving spread parameters reveal that results from the standard production model extend to the integrated model unconditionally. But the extension of standard savings model results requires a restriction on production technology.

APPENDIX

Proof of Lemma 1: Under CARA $E[U_2'']$ can be written as $-\bar{A}E[U_2']$ where $\bar{A}$ is the Arrow-Pratt measure of aversion; thus, under CARA, equation (6a) implies that:

$$C_{xs} = (1 + r)(-\bar{A})E[U_2']\{pF_x\tilde{\varepsilon} - w\} = 0 .$$

Now assume that preferences exhibit DARA and define ε^* such that $z^* \equiv pF_X\varepsilon^* - w = 0$. Assume output risk is multiplicative, implying $F_\varepsilon > 0$ and $F_{X\varepsilon} > 0$. Consider the case when $\varepsilon > \varepsilon^*(\varepsilon < \varepsilon^*)$:

$$y_2(\varepsilon) \equiv pF(X,\varepsilon) - wX + (1+r)S > (<)\, pF(X,\varepsilon^*) - wX + (1+r)S \equiv y_2^*(\varepsilon^*) \tag{A1}$$

Due to decreasing absolute risk aversion:

$$A(y_2(\varepsilon)) \equiv -U_2''(y_2(\varepsilon))/U_2'(y_2(\varepsilon)) < (>)\, A(y_2^*(\varepsilon^*)) \tag{A2}$$

Multiplying both sides of equation (A2) by $-U_2'(y_2(\varepsilon))\,(z^*) < (>)\, 0$ yields:

$$U_2''(y_2(\varepsilon))(z^*) > -A(y_2^*)U_2'(y_2(\varepsilon))(z^*). \tag{A3}$$

Thus, equation (A3) holds for all values of ε; taking expectations of both sides of equation (A3) yields:

$$E[U_2''(z^*)] > -A(y_2{}^*)E[U_2'(y_2(\varepsilon))(z^*)] = 0 \tag{A4}$$

where the last equality follows from equation (6a). □

Proof of Proposition 1: By Lemma 1, $C_{xs} = 0$ under CARA. Thus, from equation (8), the comparative static derivatives for Model C are as follows:

$$S_{c_i}^* = -(C_{ss})^{-1}\, C_{sc_i} \text{ and } X_{c_i}^* = -(C_{xx^T})^{-1}\, C_{xc_i} \tag{A5}$$

Now note that (i) C_{ss} and C_{sc_i} equal their Model B analogs, B_{ss} and B_{sb_i} evaluated at $\tilde{M} \equiv pF(X^*\,\tilde{\varepsilon}) - w^TX^*$ and (ii) C_{xx^T} and C_{xc_i} equal their Model A analogs, A_{xx^T} and A_{xa_i} evaluated at $N \equiv (1+r)S^*(\beta)$. Thus, the Proposition follows from equation (A5). □

Proof of Lemma 2: (i) $C_{s\gamma} = E[U_2''\tilde{e}] = Cov(U_2'',\tilde{e}) > 0$, where the last inequality follows from $U''' > 0$ under non-increasing absolute risk aversion (NIARA).

(ii) $C_{x\gamma} = pF'E[U_2'\tilde{e}] + pFE[U_2''\{pF'\tilde{\varepsilon} - w\}\tilde{e}]$. Under risk aversion, $E[U_2'\tilde{e}] = Cov(U_2',\tilde{e}) < 0$. Now consider the term $E[U_2''\{pF'\tilde{\varepsilon} - w\}\tilde{e}]$.

Since $\tilde{\varepsilon} = \phi + \gamma\tilde{e}$, $\tilde{e} = \dfrac{\tilde{\varepsilon} - \phi}{\gamma} = \dfrac{1}{\gamma pF'}\,[(pF'\tilde{\varepsilon} - w) + (w - \phi pF')]$. Thus:

$$E[U_2''\{pF'\tilde{\varepsilon} - w\}\tilde{e}] = \frac{1}{\gamma pF'}\,\{E[U_2''(pF'\tilde{\varepsilon} - w)^2] + (w - \phi pF')E[U_2''(pF'\tilde{\varepsilon} - w)]\} \tag{A6}$$

$E[U_2''(pF'\tilde{\varepsilon} - w)^2]$ is clearly negative under risk-aversion. Further, $E[U_2''(pF'\tilde{\varepsilon} - w)]$ is non-negative under NIARA by Lemma 1 and from equation (6a), $(w - \phi pF') = pF'\gamma E[U'_2\tilde{e}]/E[U_2^{*\prime}] < 0$ under risk aversion. The proof now follows from equation (A6). □

Proof of Proposition 2a: It follows from (10a) and (11a) that $X_\phi^* > 0$, will hold under DARA in Model C if and only if the following inequality is satisfied:

$$-\{U_1''F'E[U_2'] + U_1''FE[U_2''z] + (1+r)^2F'E[U_2']E[U_2^{*\prime\prime}]\} > 0 \tag{A7}$$

Since $U_1'' < 0$ and $E[U_2''z] > 0$ (by Lemma 1), all terms in the brackets of equation (A7) are negative; therefore, inequality (A7) is satisfied unconditionally.

Similarly, $X_\gamma^* < 0$, will hold in Model C under DARA if and only if the following inequality is satisfied:

$$\{U_1'' + (1+r)^2 E[U_2'']\}\{F'E[U_2^{*\prime}\tilde{e}] + FE[U_2'' z\tilde{e}]\} - (1+r)^2 FE[U_2''\tilde{e}]E[U_2'' z] > 0 \quad \text{(A8)}$$

Since $E[U_2'' z\tilde{e}] < 0$ (Lemma 2), the following condition is sufficient for the inequality in equation (A8) to hold:

$$E[U_2'']E[U_2'' z\tilde{e}] - E[U_2''\tilde{e}]E[U_2'' z] = pF'\gamma\{E[U_2'']E[U_2''\tilde{e}^2] - (E[U_2''\tilde{e}])^2\} > 0 \quad \text{(A9)}$$

where the equality in equation (A9) follows from substitution for z and $\tilde{\varepsilon}$. To sign the right hand side of equation (A9), define $s = (-U_2'')^{1/2}$ and $t = (-U_2'')^{1/2}\tilde{e}$; therefore:

$$E[U_2'']E[U_2''\tilde{e}^2] - (E[U_2''\tilde{e}])^2 = E[s^2]E[t^2] - (E[st])^2 > 0 \quad \text{(A10)}$$

where the inequality in equation (A10) follows from the Cauchy-Schwarz inequality.

Proof of Proposition 2b: From equation (10b) it follows that the necessary condition for extension of the result $S_\phi^* < 0$ to Model C is given by the inequality:

$$FF'\{E[U_2'']E[U_2'' z^2] - (E[U_2'' z])^2\} + FF''wE[U_2']E[U_2'']$$
$$- (F')^2 E[U_2']E[U_2'' z] > 0 \quad \text{(A11)}$$

Substituting for z rewrite equation (A11) as:

$$FF'\{E[U_2'']E[U_2'' z^2] - (E[U_2'' z])^2\} - p(F')^3 E[U_2']E[U_2''\tilde{\varepsilon}]$$
$$+ wE[U_2']E[U_2'']\{FF'' + (F')^2\} \quad \text{(A12)}$$

Since $\tilde{\varepsilon}$ has support on R_+, $E[U_2''\tilde{\varepsilon}] < 0$, and by Cauchy-Schwarz inequality the term within { } is positive; thus the expression in equation (A12) is > 0 if $\{FF'' + (F')^2\} < 0$, which is the sufficient condition for $S_\phi^* < 0$. □

From equation (12b) it follows that the necessary condition for extension of the result $S_\gamma^* > 0$ to Model C is given by the inequality:

$$E[U_2'' z]\{(F'/F)E[U_2'\tilde{e}] + E[U_2'' z\tilde{e}]\} - E[U_2''\tilde{e}]\{E[U_2'' z^2] + pF''E[U_2'\tilde{\varepsilon}]\} > 0 \quad \text{(A13)}$$

Rearranging equation (A13):

$$E[U_2'' z]E[U_2'' z\tilde{e}] - E[U_2'' z^2]E[U_2''\tilde{e}] + (F'/F)E[U_2'' z]E[U'_2\tilde{e}] - pF''E[U_2''\tilde{e}]E[U_2'\tilde{\varepsilon}] \quad \text{(A14)}$$

Substituting for $\tilde{e} = (\tilde{\varepsilon} - \phi)/\gamma = [z - (\phi pF' - w)]/\gamma pF'$ in the first two terms of equation (A14):

First Two Terms = $\frac{(\phi pF' - w)}{\gamma pF'}\{E[U_2'']E[U_2'' z^2] - (E[U_2'' z)^2]\} > 0$, where the inequality follows from Cauchy-Schwarz inequality and equation (6a). Further, substituting for $z \equiv pF'\tilde{\varepsilon} - w$ in the third term:

$$\text{Third term} = \frac{(pF')^2}{F}E[U_2''\tilde{\varepsilon}]E[U_2'\tilde{e}] - \frac{F'}{F}wE[U_2'']E[U_2'\tilde{e}]$$

$$= \frac{(pF')^2}{F} E[U_2'' \tilde{\varepsilon}] E[U_2' \tilde{e}] + w \frac{(\phi pF' - w)}{\gamma pFF'} E[U_2''] E[U_2'],$$

where the last equality follows from substituting for $\tilde{e}$ and equation (6a). Finally, in the fourth term, substituting for $\tilde{e}$ and for $E[U_2' \tilde{\varepsilon}] = wE[U_2']/pF'$ (from equation (6a)) yields:

Fourth Term = $\{-E[U_2'' z] + (\phi pF' - w)E[U_2'']\} \frac{F'' w E[U_2']}{\gamma p (F')^2}$

Adding up the terms yields:

$$\frac{(\phi pF' - w)}{\gamma pF'} \{E[U_2''] E[U_2'' z^2] - (E[U_2'' z)^2]\} + \frac{(pF')^2}{F} E[U_2'' \tilde{\varepsilon}]\, E[U_2' \tilde{e}]$$

$$- \frac{wF'' E[U_2'' z] E[U_2']}{\gamma p (F')^2} + \frac{w}{\gamma pF(F')^2} E[U_2'] E[U_2''] \{(F')^2 + F''F\} > 0 \qquad \text{(A15)}$$

Now observe: (i) The first term is positive as shown above, (ii) $E[U_2'' \tilde{\varepsilon}] < 0$ (iii) $E[U'_2 \tilde{e}] = Cov(U'_2, \tilde{e}) < 0$, and (iv) $E[U_2'' z] > 0$ (Lemma 2); hence, the condition $\{(F')^2 + F''F\} \leq 0$ is sufficient for the inequality in equation (A15) to hold. □

NOTES

1. Papers on production under uncertainty include Batra and Ullah (1974), Chavas (1985), Flacco (1983), Ishii (1977, 1989), Just and Pope (1978), Pope and Kramer (1979) and Pope (1980, 1987) and the references therein. For studies of savings and consumption choices under uncertainty, see Leland (1968), Levhari and Srinivasan (1969), Hahn (1970), Sandmo (1970), Mirman (1971), Lippman and McCall (1981) and the references therein.
2. The following notations are used throughout this paper. If $h(x,y)$ is a real-valued function of two vectors x and y, then h_x denotes the vector of partial derivatives of $h(\cdot)$ with respect to x, and h_{xy} the matrix whose ijth element is $\partial^2 h(\cdot)/\partial x_i \partial y_j$. The transpose of a matrix M is denoted by M^T;I_n indicates the $n \times n$ identity matrix, and o the null matrix of the appropriate dimension. $\mathbf{R}^n$ denotes the n-tuple of real numbers, while $\mathbf{R}^n_+$ denotes the n-tuple of real numbers in the non-negative orthant.
3. We are indebted to an anonymous reviewer for the proof of Proposition 2b.

REFERENCES

Batra, R.N., and Ullah, A. "The Competitive Firm and the Theory of Input Demand Under Price Uncertainty." *Journal of Political Economy* (May–June 1974): 537–548.

Chavas, J.P. "On the Theory of the Firm Under Uncertainty When Initial Wealth is Random." *Southern Economic Journal* (January 1985): 818–827.

Flacco, P.R. "Output, Entry and Competitive Production under Price Uncertainty." *Southern Economic Journal* (October 1983): 565–571.

Hahn, F.H. "Savings and Uncertainty." *Review of Economic Studies* (January 1970): 21–24.

Ishii, Y. "On the Theory of the Competitive Firm Under Price Uncertainty: A Note." *American Economic Review* (September 1977): 768–769.

———. "Measures of Risk Aversion and Comparative Statics of Industry Equilibrium: Correction." *American Economic Review* (March 1989): 256–286.

Just, R.E. and Pope, R.D. "Stochastic Specification of Production Functions and Economic Implications." *Journal of Econometrics* (1978): 67–86.

Leland, H.E. "Savings and Uncertainty: The Precautionary Demand for Savings." *Quarterly Journal of Economics* (1968): 465–473.

Levhari, D. and Srinivasan, T.N. "Optimal Savings Under Uncertainty." *Review of Economic Studies* (1969): 27–38.

Lippman, S. and McCall, J. "The Economics of Uncertainty: Selected Topics & Probabilistic Models," in *Handbook of Mathematical Economics*, Vol. 1, ed. by K. Arrow and M. Intriligator, (1981).

Mirman, L.J. "Uncertainty and Optimal Consumption Decisions." *Econometrica* (January 1971): 179–185.

Pope, R.D. "The Generalized Envelope Theorem and Price Uncertainty." *International Economic Review* (February 1980): 75–86.

———. "An Analogy Between Risk Aversion and Homothetic Production Under Uncertainty." *American Journal of Agricultural Economics* (May 1987): 378–381.

Pope, R.D. and Kramer, R.A. "Production Uncertainty and Factor Demands for the Competitive Firm." *Southern Economic Journal* (October 1979).

Sandmo, A. "The Effect of Uncertainty on Savings Decisions." *Review of Economic Studies* (July 1970): 353–360.

———. "On the Theory of the Competitive Firm Under Price Uncertainty." *American Economic Review* (March 1971): 65–73.